Rüdiger Schultz: On Unit Commitment

I

Birthplace of Unit Commitment

Step Back in Time for 101 years

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Zschornewitz

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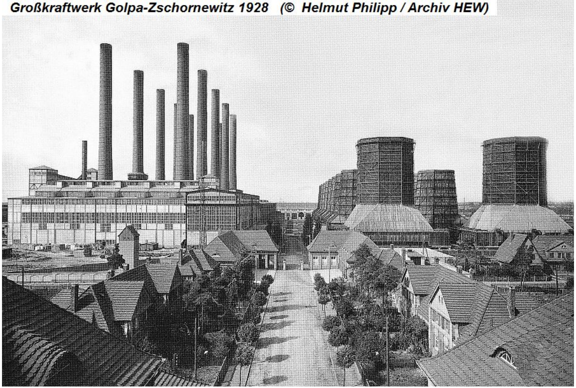

Großkraftwerk Golpa-Zschornewitz 1928 (© Helmut Philipp / Archiv HEW)

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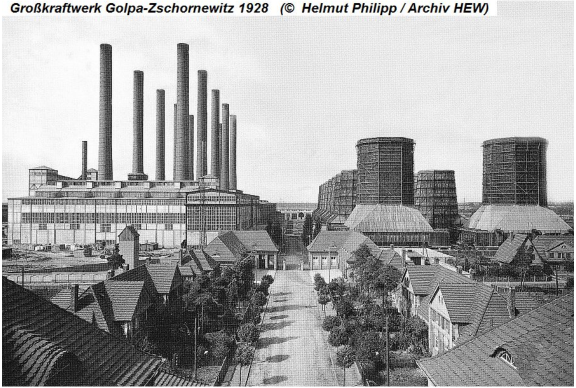

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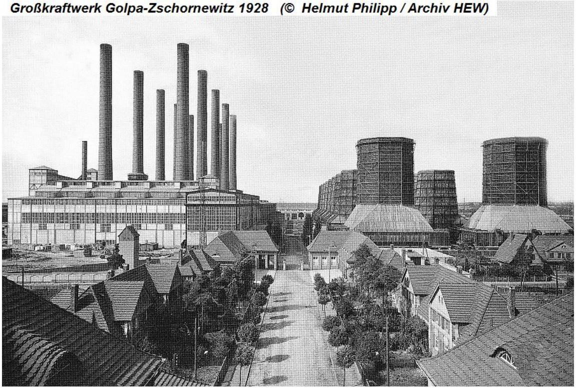

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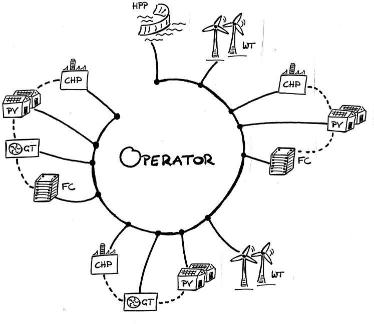

CHP : Combined heat and power units
FC : Fuel cells
GT : Gas turbines
HPP : Hydro power plants
PV : Photovoltaics
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Variables:

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Constraints:

- connecting units: load balances, reserve balances, ramping
- for individual units: output bounds, minimum up- and down-times, water management in psp,

II

The Greatest – Unit Commitment Under Uncertainty

Unit Commitment Under UNCERTAINTY

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Objective:

$f(\xi_1, .)$ random cost profile for operation and switching of thermal units inuced by start-up/shut-down scheme ξ_1

$$Q_{\mathbb{E}}(\xi_1) := \int_\Omega f(\xi_1, \omega)\, \mathbb{P}(d\omega) \text{ --- Expected Value – Risk Neutral Model}$$

Unit Commitment under Uncertainty over the Years

$$\min_x \left\{ \underbrace{c^\top x + \min_y \left\{ q^\top y \,:\, Wy = h(\omega) - Tx,\; y \in Y \right\}}_{f(x,\omega)} \,:\, x \in X \right\}$$

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- 2010: Congestion and capacity management under uncertain in- and outputs

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III

Unit Commitment – Subjective Comments

Unit Commitment and Economic Dispatch

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In recent years, the integration of UC into energy optimization models which, themselves, already are large-scale, e.g., power flow or uncertainty management in production and trading, became a focal research topic.

The Early Days

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The computationally more demanding rigorous methods, on the other hand, yield provably optimal solutions or at least lower bounds allowing for gap estimates between objective function values of the best feasible solution found so far and lower bounds generated in the course of the algorithm.

Lagrangian Relaxation - as it was

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[ShebleFahd1994] grant a “clear consensus presently tending toward the Lagrangian Relaxation (LR) over other methodologies”. Indeed, still today LR offers flexible possibilities for relaxing constraints complicating the model, however, at the cost of having to solve repeatedly “close cousins” to the relaxed problem.

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(i) relaxation of constraints interlinking units, e.g., load coverage or reserve requirements, and arrival at single-unit subproblems,

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(iv) application of Lagrangian heuristics to obtain "promising" feasible primal solutions from the results of the dual optimization.

Lagrangian Relaxation - as it is

Fueled by improved bundle-trust subgradient methods for the Lagrangian dual and by permanent progress in “off-the-shelve” mixed-integer linear programming (MILP) software, up to the advent of market deregulation, two basic aproaches developed which still today are widely used:

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(i) LR, often in conjunction with heuristic methods for finding “promising” feasible solutions,
(ii) direct solution (by branch-and-bound) of MILP formulations of UC by “off-the-shelve” solvers.

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Rather than the transition to different time horizons, from short via medium to long-term, the new economic environment in the course of energy market deregulation poses research necessities and provides incentive to integrate UC and ED with load flow and uncertainty treatment, [Gabriel-etal2013].

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Rather than the transition to different time horizons, from short via medium to long-term, the new economic environment in the course of energy market deregulation poses research necessities and provides incentive to integrate UC and ED with load flow and uncertainty treatment, [Gabriel-etal2013].

The latter is intended in the widest sense, from handling stochasticity to topics of mathematical equilibria in the context of power trading and bidding into power markets. In particular, this means to integrate UC into models which already are complex themselves.

Power Flow - Integrating UC and AC Load Flow

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This was considered utopic throughout the “Early Days”, but now became possible by studying the quadratic nonconvex AC load flow equations from the viewpoint of semidefinite optimization. In [LavaeiLow2011] after relaxation of the rank condition the solution to the dual of the remaining convex model allows to retrieve a primal solution often meeting the relaxed rank condition, and thus enabling to solve nonconvex power flow optimization problems to global optimality.

Power Flow – DC Model and Ohmic Losses

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The DC Load Flow Model provides a linear approximation of its AC counterpart by resorting to linear relations and avoiding variables in the space of complex numbers, see [Franketal2012a,Franketal2012b]. The Ohmic Losses approximation, [Sanchez-MartinRamos1997], provides the possibility to include power losses within the DC-approximation of an AC power system. Precise modeling of power losses turns out instrumental in congestion management when load dispatches or even commitments of units have to be revised to increase throughput of the grid under increased inflows of renewables.

Polyhedral Methods

Polyhedral Methods

Despite its success in combinatorial optimization, cutting plane methods based on polyhedral studies, either applied directly or enhancing branch-and-bound came to the fore in UC a bit more than 10 years ago, only. `There always have been polytopes with fancy names ''pretending to be real-life'' . However, UC, really a real-life model in this respect, was a no-name product, which has changed, fortunately.` At this time, market deregulation enforced the need of solving UC in a competitive environment under incomplete information. In this way, solving UC problems became a subroutine in the treatment of more complex decision problems in electricity supply. Today tight formulations for crucial model ingredients and for complete polytopes arising in UC are available [LeeLeungMargot2004,Morales-EspanaGentileRamos2015].

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After deregulation UC-relevant sources of uncertainty have spread considerably: power input from renewables, power prices determined by bidding into power exchanges, competitors' actions at electricity markets.

Yet, UC is understood in a broader context than before. It rather is the scheduling of decentralized power supply with its small generating facilities than commitment of thermal let alone nuclear generation units.

While the mathematical apparatus is fairly well developed for exogenous uncertainty, the situation is completely different for endogenous uncertainty, i.e., with decision dependent probability distributions.

In case uncertainty is captured by probability measures, stochastic integer programming offers methodology for handling UC, both algorithmically and regarding structural understanding, [TakritiBirgeLong1996,CarøeSchultz1998,Schultz2003].

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TABLE I
COMPARISON OF SUC METHODS

Stochastic Optimization UC methods	Structures	Algorithms and Literature	Advantages	Disadvantages
Stochastic Programming	Two-stage models	▪ Benders Decomposition (BD) [16][55][78] ▪ Accelerated BD [99] ▪ Lagrangian Relaxation (LR) [56][73] ▪ Stabilized LR (using Bundle methods) [47-49][57] ▪ Sample Average Approximation [74-76]	▪ Minimize total expected cost; easier to understand (and compute) than minimizing regret or minimizing the worst-case cost. ▪ Various decomposition and sampling-based algorithms already existed with convergence and performance guarantees. ▪ Can address robustness issues using risk measures. ▪ Can provide expected value of perfect information (EVPI) and value of stochastic solution (VSS).	▪ Need to assign probabilities for scenarios. ▪ Computationally demanding for large numbers of scenarios. ▪ Difficulties in dealing with integer variables in the second stage (e.g., unit rescheduling in real-time). ▪ Static assumption of the uncertainties.
	Multi-stage models	▪ Lagrangian Relaxation (LR) [42-43] ▪ Augmented LR [45] ▪ Column Generation (CG) [65] ▪ Progressive Hedging [44] ▪ Stabilized LR or CG (using Bundle methods) [48][50][64] ▪ Nested CG [67]	▪ Truly a decision-making model (as opposed to "what-if" analysis) over multiple time periods under uncertainty. ▪ Ability to model the dynamic process of uncertainties and decisions. ▪ Useful for systems with generators that can reschedule quickly. ▪ Can provide EVPI and VSS.	▪ Curse of dimensionality, and hence computationally very expensive. ▪ Need explicit scenario trees and random paths' probabilities. ▪ Even more difficult with integer variables present in all stages.
Robust Optimization	Bi-level and tri-level models	▪ Benders Cutting Plane method (dual) [27][29][32-35][87-89] ▪ Column-Constraint Generation method (primal) [28][34]	▪ Do not need probability distribution. ▪ Solutions can provide decision-makers guarantee towards the worst-case. ▪ Computationally not as demanding as stochastic programming models with large numbers of scenarios.	▪ May yield over-conservative solutions. ▪ Need expertise and rationale on uncertainty set construction. ▪ Need to use different algorithms for different types of uncertainty sets. ▪ Difficult to incorporate the uncertainty dynamics (e.g., multi-level/stage models).
(Approx.) Stochastic Dynamic Programming	multi-stage, discrete time models	▪ Value-function approximation [97] ▪ Policy iteration/Model predictive control [95, 96] ▪ State-space approximation [94]	▪ ADP can handle multi-stage stochastic problems with relatively low computational burden. ▪ Can model closed-loop systems (such as real-time pricing).	▪ For ADP, convergence to optimal solutions may be difficult to establish. ▪ Integer variables may present difficulties in general.