On time-consistent and -inconsistent stochastic dominance risk averse measures for Capacity Expansion Planning in electricity generation systems and transmission networks along a time horizon and related SDP decomposition algo plus HPC

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- Time-inconsistent Stochastic Dominance (TSD) risk averse measure
- Time-consistent Expected Stochastic Dominance (ESD) risk averse measure
- Types of decomposition methods for stochastic optimization
- Brief ref. to some decomposition methods for multistage stochastic scenario tree node cross constraints mixed 0-1 risk averse problems
- Stochastic Dynamic Programming decomposition matheuristic algorithm
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- **AIM AND SCOPE**

Aim

- Presenting models for addressing challenges for a long term (e.g., 30 years time horizon) capacity expansion planning in electricity generation and transmission networks along the years of the time horizon.
- Scope: Helping to decide on:
  1. GEP: Type and mix of power generation sources (ranging from less coal, nuclear and combined cycle gas turbine to more renewable sources: hydroelectric, wind, solar, photovoltaic, fossil fuels and biomass)
  2. GEP: New power generation plant / farm location and capacity
  3. NEP: Location and capacity of new lines in the transmission network
- By considering a variety of risk averse measures for risk management, and
- Using a SDP matheuristic algo for problem solving.

Goals to achieve. Helping on quantifying

- GEP: Satisfying electricity demand in the service network of the GenCo, and Maximizing different types of utility criteria:

  Benefits of using cleaner, safer and efficient (cheaper) energy accessible to all the consumption nodes in the network.
- NEP: Eliminating existing technological and political barriers

Contents

- **ELECTRICITY GENERATION CAPACITY EXPANSION PLANNING (GEP): MULTISTAGE STOCHASTIC MIXED 0-1 MODELING**

GEP. Characteristics

- Uncertainty, Multicriteria and Nonlinearity
- A gigantic but well structured multicriteria multistage SMINO problem with risk management.
  - Dynamic setting
  - Site location and capacity decisions
  - Current and candidate power generation plants / farms energy transmission nodes, demand nodes)
  - Replicated networks (hydro valleys). E.g., EdF 50+ valleys, some with 50 elements, see Charousset, COST WMINLP, Paris, 2013.
- Algorithmic framework for MINO under uncertainty in dynamic setting, see LFE et al., WMINLP, Pittsburgh, 2014.
- SMINO in Electricity Generation, see Charousset, COST WMINLP, Paris, 2013.

Uncertainty in GEP

- A stage in time horizon: Consecutive years whose constraint systems must be satisfied in an individual basis.
- A gigantic multistage [non-symmetric] scenario tree. E.g., Brazilian power system: 120 periods, 20^{119} scenarios (Sagastizabal MP'12).
- It is **required** a combination of:
  - Sample scenario schemes
  - SMINO $\rightarrow$ Sequential SMILO
  - inexact node-based Decomposition algos
  - High Performance Computing

Multicriteria in GEP

- Maximizing NPV of expected investment (and consumer stakeholders) goals over the scenarios along the time horizon subject to risk reduction of the negative impact of non-wanted scenarios on multiple types of utility objectives and stakeholders:
    - Maximizing power share of cleaner, safer and efficient -cheaper- energy accessible to all consumption nodes.
    - Generation Network reliability.
    - EC directives on environmental issues and others.
    - EU governments, etc.

Some facts on European Renewable Energy Sources (RES) Generation systems

- EU has established aggressive emission reduction targets: a 20 % (res. 27 %) reduction in greenhouse gases with respect to 1990 levels by 2020 (res. 2030) (most of the member countries are still far away from that targets) and endorsing an objective of 80 % reductions by 2050.

  See epp.eurostat.ec.europa.eu/portal/page/portal/eurostat/home/.
- So, vast amounts of new generation plants / farms are expected to be built in the medium term future. A substantial part of this new RES generation could probably materialize in the near future. '

Some European projects on RES Generation systems

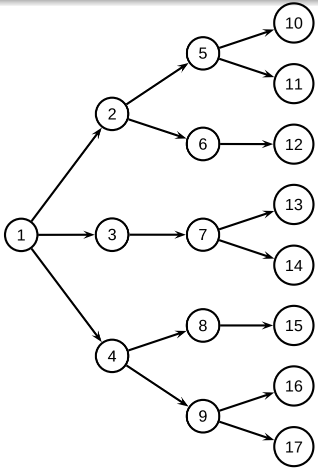
- OWF: 5.3 GW installed in Europe (including London array, 1 GW). See EWEA. Wind in power. report 2012.
- See Achim Woyte and 3E OffshoreGrid. An Intelligent Energy Europe Project, 2009.
- London Array. See www.londonarray.com.
- NSCOGI. See www.The North Sea Countries' Offshore Grid Initiative.

Deterministic mixed 0-1 optimization model

$$
\begin{aligned}
z_{EV} = & \max \sum_{t \in \mathcal{T}} (a^t x^t + b^t y^t) \\
\text{s.t.} & \sum_{t' \in \mathcal{A}^t} (A_t^{t'} x^{t'} + B_t^{t'} y^{t'}) = h^t \quad \forall t \in \mathcal{T} \\
& x^t \in \{0,1\}^{nx(t)}, y^t \in \mathbb{R}^{ny(t)} \qquad \forall t \in \mathcal{T}.
\end{aligned} \tag{1}
$$

Math optim under uncertainty

- **Multistage scenario tree**.
- A **stage** of a given horizon is a set of consecutive time periods (i.we., years) where the realization of the uncertain parameters takes place.
- A **scenario** is a realization of the uncertain parameters along the stages of a given horizon.
- A **node** for a given stage in the scenario tree has a 1-1 correspondence with the group of scenarios that have the same realization of the uncertain parameters up to the stage.
- **Nonanticipativity principle** (Wets SIAM Review'74): The scenarios of a group have a unique solution for the stage which correspondent node belongs to.



$\Omega = \Omega^1 = \{10, 11, \ldots, 17\}$; $\Omega^2 = \{10, 11, 12\}$

$\mathcal{G} = \{1, \ldots, 17\}$; $\mathcal{G}^2 = \{2, 3, 4\}$

$\mathcal{A}^{17} = \{1, 4, 9, 17\}$

Figura: Multistage nonsymmetric scenario tree

Scenario tree notation

- $\mathcal{E}$, set of the stages along the horizon. Note: $E = |\mathcal{E}|$.
- Ω, set of scenarios.
- $\mathcal{G}$, set of nodes in the tree.
- $\Omega^g \subseteq \Omega$, set of scenarios in a group with a 1-1 correspondence to node g, for $g \in \mathcal{G}$.
- $\mathcal{G}^e \subset \mathcal{G}$, set of nodes for stage e, for $e \in \mathcal{E}$.
- $e(g)$, stage to which node g belongs to, for $g \in \mathcal{G}$.
- w^ω, weight or probability assigned to scenario $\omega \in \Omega$.
- w^g, weight or probability assigned to node $g \in \mathcal{G}$. It is computed as $w^g = \sum_{\omega \in \Omega} w^\omega$
- $\mathcal{F}$, set of functions / criteria to be satisfied, such that function indexed by $f = 1$ is the objective fun to maximize.

Scenario tree notation (c.)

$\tilde{A}^g$, set of ancestor nodes to node g (including itself), for $g \in \mathcal{G}$.

$\mathcal{A}^g \subseteq \tilde{A}^g$, set of ancestor nodes to node g in the scenario tree (including itself) with nonzero elements in constraints of node g, for $g \in \mathcal{G}$.

$\beta(g)$, immediate ancestor node in the scenario tree of node g, for $g \in \mathcal{G}$. Let us assume that $\beta(1)$ is empty.

$\mathcal{S}^g$, set of successor nodes in the scenario tree to node g (excluding itself) (i.e., nodes in the subtree whose root is node g), for $g \in \mathcal{G}$,

Note: Set of scenarios Ω^g of any leaf node g (i.e., last node) is singleton. Let us assume that $\omega = g$ for $\omega \in \Omega^g$ and $g \in \mathcal{G}^E$.

$\mathcal{S}_1^g \subseteq \mathcal{S}^g$, set of immediate successors of node g, for $g \in \mathcal{G} : e(g) < T$.

Scenario tree notation (c.)

$\mathcal{T}$, set of periods (usually, years) in the time horizon. Last period $T = |\mathcal{T}|$.

$\mathcal{T}^e$, set of (consecutive) periods in stage e, for $e \in \mathcal{E}$, $\mathcal{T}^e \cap \mathcal{T}^{e'} = \emptyset$, $e, e' \in \mathcal{E} : e \neq e'$, $\mathcal{T} = \cup_{e\in\mathcal{E}} \mathcal{T}^e$.

$t(e)$, first period in set $\mathcal{T}^e$, for $e \in \mathcal{E}$.

Let (t, g) denote the pair of indexes for period t and node g, for $t \in \mathcal{T}^{e(g)}, g \in \mathcal{G}$.

A gigantic stochastic problem

- A stage in time horizon: Consecutive years whose constraint systems must be satisfied in an individual basis.
- A multistage stochastic [non-symmetric] scenario tree. E.g., Brazilian power system: 120 periods, 20^{119} scenarios (Sagastizabal MP'12).
- So, it is **required** a combination of:
  - Sample scenario schemes
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Example of four-stage scenario tree (15 nodes, 8 scenarios)

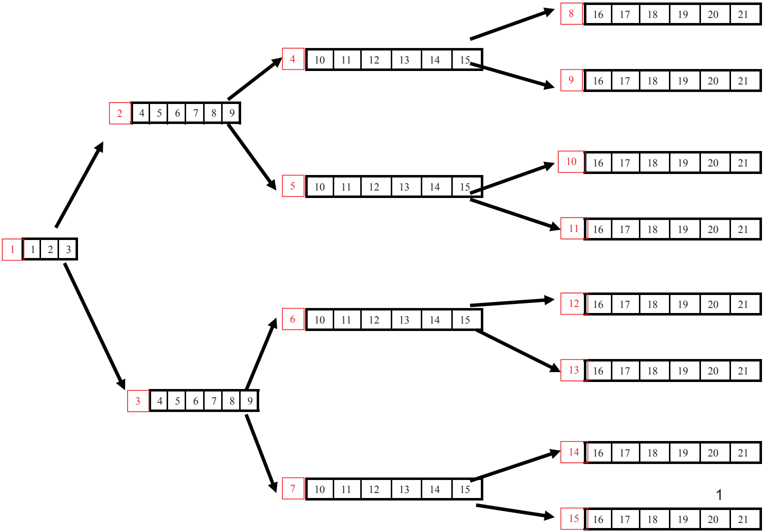

Figura: Vespucci CMS'11

Risk Neutral (RN). Parameters

- a_1^g, $b_1^{t,g}$, vectors of objective function coeffs for variables in vectors $x^g, y^{t,g}$, res.
- $h^{t,g}$, rhs for set of constraints related to node g in period t, for $t \in \mathcal{T}^{e(g)}$, $g \in \mathcal{G}$.
- $A_{t,g}^{g'}$, $B_{t,g}^{t',g'}$, matrices related to ancestor node g' and ancestor pair (t', g') in constraints related to pair (t, g), res., for $t' \in \mathcal{T}^{e(g')} : t' \leq t, g' \in \mathcal{A}^g, t \in \mathcal{T}^{e(g)}, g \in \mathcal{G}$.

  Notice that $t' < t$ for $g' \neq g$.

GEP. Risk Neutral (RN). Vector of variables x^g in node $g \in \mathcal{G}$ of the scenario tree

A vector of 0-1 variables for defining the power generation capacity expansion, such that the value 1 for each element means that its construction starts in the (first period of) stage $e(g)$ for node g and otherwise, 0. A related element can be:

- generation plant, wind farm, solar farm, photovoltaic farm of a given candidate technology,
- hydro turbine

A vector of continuous variables, such that each element represents for period t in node g one of the following elements:

- Energy generated by the power plants / farms,
- water stored in the reservoirs, water released through the canals in the hyper-hydro valleys,
- Green Certificates sold or bought by the generators, CO_2 that is emitted,
- storage, supplying and consumption of the raw materials (gas, etc.), etc.

GEP. RN multistage mixed 0-1 DEM. Compact representation

Risk neutral model that synthesizes model (5)-(21) (see below):

$$z_{RN} = \text{máx} \sum_{g \in \mathcal{G}} w^g \left(a_1^g x^g + \sum_{t \in \mathcal{T}^{e(g)}} b_1^{t,g} y^{t,g} \right) \tag{2}$$

subject to

$$\sum_{g' \in \mathcal{A}^g} \sum_{t' \in \mathcal{T}^{e(g')} : t' \leq t} (A_{t,g}^{g'} x^{g'} + B_{t,g}^{t',g'} y^{t',g'}) = h^{t,g} \quad \forall t \in \mathcal{T}^{e(g)}, \, g \in \mathcal{G}$$

$$x^g \in \{0,1\}^{nx(g)}, \, y^{t,g} \in \mathbb{R}^{ny(t,g)} \quad \forall t \in \mathcal{T}^{e(g)}, \, g \in \mathcal{G}$$

Note: The constraint qualification $t' \leq t$ is only active for $g' = g$.

GEP. Types of constraints

For the periods along the time horizon over the scenarios, satisfying automatically the NAC (since the above representation of the model is a compact one) lower / upper bounds on:

- Electricity generated from each available current power plant / farm and those of the candidate technologies that being previous started their construction are already available,
- Raw material availability for the thermal plants,
- Replicated network definition of the reservoirs for the hyper-hydro valleys,
- Electricity generated from each available current and potential new turbines, if they are available,
- Periodic repayment of investment cost allocation of new power plants / farms,
- Green Certificates and CO_2 emission definition and bounding,

GEP. Modeling

- Approximation: A huge but very well structured multicriteria multistage stochastic mixed 0-1 linear optimization problem
- GenCo wishes to determine its optimal planning for investment in power generation capacity in a long term horizon.
- Regulatory Authorities aim: Promoting the development of RES (i.e. by hydro, wind, solar, photovoltaic, fossil fuels and biomass power plants) for power generation systems with reduced CO_2 emissions and penalization.

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GEP. Description

- Green Certificate schemes support power generation from RES, and penalizes generation from conventional power plants (e.g., CCGT, coal, nuclear power plants).
- Every year a prescribed ratio is required between the electricity generated from RES and the total generated.
- In case the actual ratio attained at a given year does not exceed the prescribed one, the energy generator has to buy Green Certificates in order to satisfy the related constraint.
- On the contrary, when the actual ratio attained is greater than the prescribed one, the energy generator can sell Green Certificates in the market

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GEP. Description (c.)

- GenCo's aim: Maximizing expected profit along the time horizon at NPV subject to feasible constraints for nodes (i.e., scenario groups) in the scenario tree.
- Revenues from sale of electricity depend on the market price and the amount of electricity sold, which is bounded above by market competition. The revenues also depend on the number of operating hours per year of the power plants / farms in the generation system.
- Penalization for emitting CO_2 greatly varies among generation technologies.
- Variable and fixed costs also greatly differ among the generation technologies.
- Investment costs on some types of generation technologies depend on the plant rated power and on the investment costs per power unit.

- Revenues and costs associated to the Green Certificate scheme depend on the Green Certificate price as well as on the yearly ratio between generation from RES and total annual generation.
- The evolution of electricity prices in nodes covered by the energy network along the time horizon is not known at the time when the investment decisions are to be made.

  Therefore, a risk is associated with the expected profit from power generation capacity and energy transmission expansion, due to the uncertainty on the main parameters.
- Our risk management proposal: A mixture of time-inconsistent and time-consistent measures, such as Conditional Value-at-Risk (TCVaR / ECVaR) or stochastic dominance (TSD/ESD) measures.

- The proposed model determines the evolution of the power generation mix and location along the time horizon. So, it determines for every power generation technology technology:
  - **site location of each power plant / farm**
  - **year to start the construction**

  depending on the node in the scenario tree (i.e., scenario group) along the time horizon.
- It could be possible that at the year when the new power plant is ready for being in operation, **the realization of uncertain prices and electricity demand and other key uncertain elements be drastically changed along the scenario tree from the node in the tree where the construction have started.**

Example of four-stage scenario tree (15 nodes, 8 scenarios)

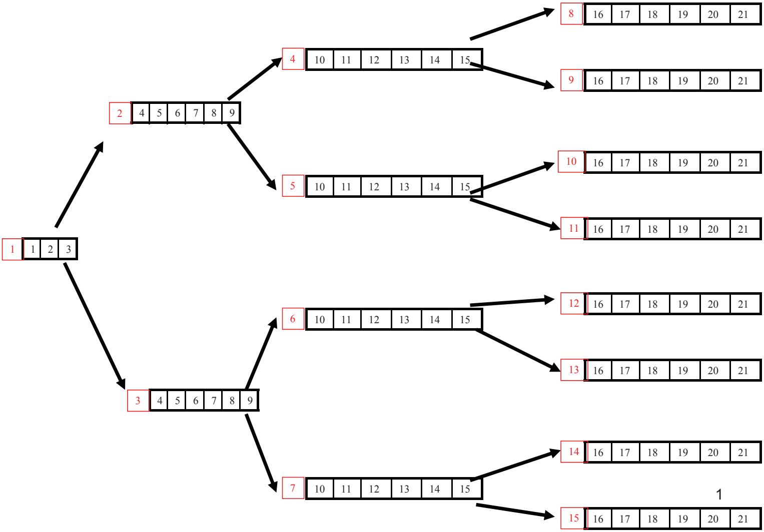

Figura: Vespucci CMS'11

GEP. SMILP approximating model due to its SMINLP complexity and long time horizon

- Linear functions approximate nonlinearities on:
  - Investment and operation costs for generation and transmission
  - Hydropower generation
  - Windpower solarpower generation,
  - Generation losses, etc.
- Unit commitment (scheduling problem) has been over simplified in the model.

GEP. Problem modeling. Power generation sets

$\mathcal{N}^T$, Candidate technologies for thermal power generation.

$\mathcal{N}^R$, Candidate technologies for power generation from RES (without considering power generation from hyper hydro valleys since they have a different treatment).

$\mathcal{N}$, Candidate technologies (i.e., $\mathcal{N} = \mathcal{N}^T \cup \mathcal{N}^R$).

$\mathcal{I}^{n^T}$, Set of potential thermal power plants of technology n, for $n \in \mathcal{N}^T$ that can be constructed.

$\mathcal{I}^{N^T} = \cup_{n \in \mathcal{N}^T} \mathcal{I}^{n^T}$, Set of potential thermal power plants.

$\mathcal{I}^{n^R}$, Set of potential power RES plants / farms of technology n, for $n \in \mathcal{N}^R$ that can be constructed.

$\mathcal{I}^{N^R} = \cup_{n \in \mathcal{N}^R} \mathcal{I}^{n^R}$, Set of potential RES power plants / farms (without considering hyper hydro valleys).

$\mathcal{I}^{N} = \mathcal{I}^{N^T} \cup \mathcal{I}^{N^R}$, Set potential power plants / farms.

Note: Each new plant / farm will be sited in a given already chosen location (i.e., node in the energy netowrk), if it is selected.

$n(i)$, Technology type of potential power plant $i \in \mathcal{I}^N$.

$\mathcal{I}^{K^T}$, Current thermal power plants.

$\mathcal{I}^{K^R}$, Current RES power plants / farms.

$\mathcal{I}^{K} = \mathcal{I}^{K^T} \cup \mathcal{I}^{K^R}$, Current power plants / farms (i.e., at period 0 of the time horizon)..

Note: It is assumed that the sites are unique for the currently owned and potential new plants / farms, so, $\mathcal{I}^{N} \cap \mathcal{I}^{K} = \emptyset$.

Raw material sets for thermal plants

$\mathcal{J}$, Key raw materials (fuel, gas, etc.) required by thermal power plants (current ones and plants from new technologies).

$\mathcal{J}_i \subseteq \mathcal{J}$, Raw materials required by any plant of candidate power thermal technology $i \in \mathcal{N}^K$ or current power thermal plant $i \in \mathcal{I}^K$.

$\mathcal{V}$, Hyper hydropower valleys (i.e., valleys with hyper period water stored).

$\mathcal{I}^v$, Reservoirs in hydropower valley $v \in \mathcal{V}$.

$\mathcal{U}_i \subset \mathcal{I}^v$, (Immediate) Upstream reservoirs to reservoir $i \in \mathcal{I}^v, v \in \mathcal{V}$.

$\mathcal{D}_i \subset \mathcal{I}^v$, (Immediate) Downstream reservoirs to reservoir $i \in \mathcal{I}^v, v \in \mathcal{V}$.

$\tilde{\mathcal{D}}_i \subset \mathcal{D}_i$, Reservoirs $\{j\}$, such that 'canal' ij has a potential increase of power generation from the current one (that even can be zero), for $i \in \mathcal{I}^v, v \in \mathcal{V}$.

$\mathcal{I}^V = \cup_{v \in \mathcal{V}} \mathcal{I}^v$, Reservoirs in the hydropower valleys.

Number of time periods

S_n [t], Construction time periods required for any power plant / farm of candidate technology $n \in \mathcal{N}$ to be available for power generation.

L_i [t], Industrial life of any current power plant / farm $i \in \mathcal{I}^K$.

S_{ij} [t], Construction periods required for hydro power generation turbine(s) in 'canal' ij, for $j \in \tilde{\mathcal{D}}_i$, $i \in \mathcal{I}^V$.

ρ [$-$], Discount rate.

M_i, R_i [*MEURO*], Investment cost and its periodic allocation (see below), res., required by candidate power plant / farm i, for $i \in \mathcal{I}^N$.

M_{ij}, R_{ij} [*MEURO*], Investment cost and its periodic allocation (see below), res., required by hydro power generation turbine(s) in 'canal' ij, for $j \in \tilde{\mathcal{D}}_i$, $i \in \mathcal{I}^V$.

GEP. Problem modeling. Deterministic parameters (c.)

$\overline{I}^n$ $[-]$, Maximum number of plants / farms of candidate technology $n \in \mathcal{N}$ that can be constructed along the time horizon.

P_i $[MW]$, Rated power of any plant / farm of candidate technology $i \in \mathcal{N}$ or current power plant / farm $i \in \mathcal{I}^K$.

P_{ij} $[MW]$, Rated power of the set of turbines in 'canal' ij of hydro reservoir i, for $j \in \mathcal{D}_i$, $i \in \mathcal{I}^V$.

$\underline{P}_{i,t}$ $[GWh]$, Minimum generated energy imposed for any plant / farm of candidate technology $i \in \mathcal{N}$ or current power plant / farm $i \in \mathcal{I}^K$ at period $t \in \mathcal{T}$.

GEP. Problem modeling. Deterministic parameters (c.)

ν_i [$-$], Percentage of loss of any power plant / farm of candidate technology $i \in \mathcal{N}$ or current power plant / farm $i \in \mathcal{I}^K$.

ζ_i [TM/GWh], CO_2 emission rate of current power plant $i \in \mathcal{I}^{K^T}$.

φ_{ij} [MWh/TM], Assumed constant converting one TM of raw material $j \in \mathcal{J}_i$ into energy generated by the current thermal plant i, for $i \in \mathcal{K}^T$.

σ_{ijc} [$-$] and [λ_{ijc}] [oh], Loss rate and Reactance of cable $c \in \mathcal{C}_{ij}$ of current transmission line $ij \in \mathcal{L}^K$ in the energy network, res.

$\overline{R}_{ij}^{K}$ [h^3], Water capacity for hydro power generation in current turbine(s) in 'canal' ij, for $j \in \mathcal{D}_i$, $i \in \mathcal{I}^V$.

ϕ_{ij}^{K} [MWh/h^3], Assumed constant converting one h^3 of water into energy generated by the current (identical turbine(s)) in 'canal' ij, for $j \in \mathcal{D}_i$, $i \in \mathcal{I}^V$.

GEP. Problem modeling. Uncertain parameters for any period from set $\mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$H_{i,g}$ [h], Operating hours of any power plant / farm of candidate technology $i \in \mathcal{N}$ or current power plant $i \in \mathcal{I}^K$.

$\overline{P}_{i,g}$ [GWh], Maximum electricity to generate by any power plant / farm of candidate technology $i \in \mathcal{N}$ or current power plant / farm $i \in \mathcal{I}^K$. See below.

$\varphi_{nj,g}$ [MWh/TM], Assumed constant converting one TM of raw material $j \in \mathcal{J}_i$ into electricity generated by any thermal power plant of candidate technology $n \in \mathcal{N}^T$.

$(\overline{N}_{j,g})$ [TM], Upper bound on the supply of raw material $j \in \mathcal{J}$ for the thermal power plants owned by the GenCo.

GEP. Problem modeling. Uncertain parameters for clean energy looking in any period from set $\mathcal{T}^{e(g)}$ of node $g \in \mathcal{G}$ of the scenario tree

$\overline{Q}_g$ [GMh], (negative) Lower bound on the Green Certificates. Notice that if they are negative the GenCo will pay for them; otherwise, they can be sold n the market.

γ_g [$-$], Ratio to be attained of electricity generated from RES plants plus GC and total electricity generated by the GenCo.

μ_g [TM/GWh], Upper bound on the CO_2 that can be allowed by one GWh of electricity to be generated by the total set of owned and available new thermal plants of the GenCo.

$\zeta_{n,g}$ [TM/GWh], CO_2 emission rate of any thermal power plant of candidate technology $n \in \mathcal{N}^T$.

GEP. Problem modeling. Uncertain parameters in reservoir $i \in \mathcal{I}^V$ in the hyper hydro valleys for any period from set $\mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$W_{i,g}$ $[h^3/t]$, Water exogenous inflow into reservoir i. Note: Its value is zero for run-of-the river plants.

$\overline{R}^N_{ij,g}$ $[h^3/h]$, Water capacity for hydro power generation in candidate turbine(s) in 'canal' ij, for $j \in \tilde{\mathcal{D}}_i$.

$\phi^N_{ij,g}$ $[MWh/h^3]$, Assumed con'stant converting one h^3 into electricity to be generated by the candidate turbine(s) for hydro power generation in 'canal' ij, for $j \in \tilde{\mathcal{D}}_i$.

$\underline{W}_{i,g}$, $\overline{W}_{i,g}$ $[h^3]$, Lower and upper bounds of stored water at the end of any period in reservoir i.

$\underline{R}_{ij,g}$, $\overline{R}_{ij,g}$ $[h^3/t]$, Lower and upper bounds of the release water in 'canal' ij, for $j \in \mathcal{D}_i$.

GEP. Problem modeling. Uncertain market electricity price and demand to the whole generation system $\mathcal{I}^K \cup \mathcal{I}^V \cup \mathcal{I}^N$ in period $t \in \mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$\overline{D}_{t,g}$ [GWh], demand.

$\pi^D_{t,g}$ [$kEURO/GWh$], price.

GEP. Problem modeling. Uncertain cost parameters for any period in node $g \in \mathcal{G}$

π_g^{GC} $[kEURO/GWh]$, Green Certificate price.

$\pi_g^{CO_2}$ $[kEURO/TM]$, CO_2 emission penalization.

For any thermal power plant of candidate technology $n \in \mathcal{N}^K$ (no hydro plants):

$c_{n,g}^F$ $[kEURO]$, Fixed power generation cost.

$c_{n,g}^V$ $[kEURO/GWh]$, Variable power generation cost.

For any current thermal power plant $i \in \mathcal{I}^K$ (no hydro plants):

$c_{i,g}^F$ $[kEURO]$, Fixed power generation cost.

$c_{i,g}^V$ $[kEURO/GWh]$, Variable power generation cost.

Fixed cost for:

$c_{ij,g}$ $[kEURO]$, New or current hydro turbine(s) in 'canal' ij, for $j \in \mathcal{D}_i$, $i \in \mathcal{I}^V$.

GEP problem modeling. Uncertain economic parameters for whole set of periods in $\mathcal{T}^{e(g)}$ for node $g \in \mathcal{G}$ of the scenario tree

B_g [*MEURO*], Upper bound on the total investment cost amortization that is allocated, due to the availability of the total new power plants / farms and hydro power generation turbines.

GEP. Problem modeling. 0-1 x-variables for power generation capacity expansion in the electricity network at (first) period $t(e(g))$ of stage $e(g) \in \mathcal{E}$ in node $g \in \mathcal{G}$ of the scenario tree

The value 1 for a 0-1 variable means that the entity starts its construction **by** that period and otherwise, 0.

$x_{i,g}$ [−], for potential new power plant / farm $i \in \mathcal{I}^N$, for $g \in \mathcal{G} : t(e(g)) = 1, \cdots, T - S_{n(i)}$.

$x_{ij,g}$ [−], for potential new turbine(s) in 'canal' ij for $j \in \tilde{\mathcal{D}}_i$, $i \in \mathcal{I}^V$, for $g \in \mathcal{G} : t(e(g)) = 1, \cdots, T - S_{ij}$.

Observe that, without loss of generality, it is assumed that the starting construction period of any plant / farm can only be performed at the first period of any stage along the time horizon.

GEP. Problem modeling. Continuous variables for time period $t \in \mathcal{T}^{e(g)}$ of stage $e(g) \in \mathcal{E}$ in node $g \in \mathcal{G}$ of the scenario tree

$p_{i,t,g}$ [GWh], Electricity generated by power plant / farm $i \in \mathcal{I}^N \cup \mathcal{I}^K$.

$w_{i,t,g}$ [h^3], Water stored in reservoir $i \in \mathcal{I}^V$ at the beginning of the period.

$r_{ij,t,g}$ [h^3], Water released through 'canal' ij at the period, for $j \in \mathcal{D}_i$, $i \in \mathcal{I}^V$

$q_{t,g}$ [GWh], Green Certificates sold ($q_{t,g} > 0$) or bought ($q_{t,g} < 0$) by the GenCo.

$o_{t,g}$ [TM], CO_2 generated in period t (and, then, to be paid for) by the GenCo.

GEP. Problem modeling. Continuous variables for time period $t \in \mathcal{T}^{e(g)}$ of stage $e(g) \in \mathcal{E}$ in node $g \in \mathcal{G}$ of the scenario tree (c)

For raw material $j \in \mathcal{J}$:

$e_{j,t,g}$ [TM], Storage of the raw material at the beginning of the period.

$n_{j,t,g}$ [TM], Supplying of the raw material at the period.

$m_{ij,t,g}$ [TM], Consumption of raw material for the energy generated by thermal power plant $i \in \mathcal{I}^{K^T} \cup \mathcal{I}^{N^T}$.

For electricity demand served by the whole generation system $\mathcal{I}^K \cup \mathcal{I}^V \cup \mathcal{I}^N$:

$s_{t,g}$ [GWh].

GEP. Problem modeling. Risk Neutral objective function

Maximizing the NPV of the expected profit minus the fixed generation cost of current hydro power turbines of reservoirs, current thermal power plants / farms:

$$z = \max \sum_{\omega \in \Omega} w^{\omega} z^{\omega} - \sum_{g \in \mathcal{G}} w^{g} \frac{1}{(1+\rho)^{t(e(g))}} \Big(\sum_{i \in \mathcal{I}^V} \sum_{j \in \mathcal{D}_i \setminus \tilde{\mathcal{D}}_i} c_{ij,g} + \sum_{i \in \mathcal{I}^K : i \leq L_i} c^F_{i,g} \Big), \quad (3)$$

where z^{ω} gives the NPV of profit (4) under scenario $\omega \in \Omega$, such that max z (3) is subject to constraints (5)-(21).

GEP. Problem modeling.
Risk Neutral objective function

Elements of NPV of profit z^ω under scenario $\omega \in \Omega$:

- Revenue from sale of electricity.
- Revenue from sale (or, alternatively, cost from purchase) of Green Certificates.
- Penalization of CO_2 emission.
- Variable generation cost of thermal power plants.
- Variable generation cost of RES power plants, without including hydro generation plants.
- Periodic debt repayment of all the new power plants / farms and new hydro power turbines.
- Fixed power generation cost of available new plants / farms and new hydro power turbines.

z^ω function

$$
\begin{aligned}
z^\omega = & \sum_{g\in\mathcal{A}^\omega} \sum_{t\in\mathcal{T}^{e(g)}} \frac{1}{(1+\rho)^t} \\
& \big[\pi^D_{t,g} s_{t,g} + \pi^{GC}_g q_{t,g} - \pi^{CO_2}_g o_{t,g} \\
& - \sum_{i\in\mathcal{I}^N} c^V_{n(i),g} p_{i,t,g} - \sum_{i\in\mathcal{I}^K} c^V_{i,g} p_{i,t,g}\big] \\
& - \sum_{g\in\mathcal{A}^\omega} \frac{1}{(1+\rho)^{t(e(g))}} \big[\sum_{i\in\mathcal{I}^N} (R_i + c^F_{n(i),g}) x_{i,g'} \\
& \qquad + \sum_{i\in\mathcal{I}^V} \sum_{j\in\tilde{\mathcal{D}}_i} (R_{ij} + c_{ij,g}) x_{ij,g''}\big]
\end{aligned}
\tag{4}
$$

where $g' \in \mathcal{A}^g : t(e(g')) = t(e(g)) - S_{n(i)}$ (res., $g'' \in \mathcal{A}^g : t(e(g'')) = t(e(g)) - S_{ij}$) is the ancestor node in the scenario tree to node g, its value 1 of variable $x_{i,g'}$ (res., $x_{ij,g''}$) defines that the related entity has started its construction **by** period $t(e(g))$ and, then, the entity is available for production.

GEP. Defining the 0-1 character of the x-step variables for the plants / farms of candidate technology $n \in \mathcal{N}$ that can started its construction **by** the (first) period $t(e)$ of stage $e(g) \in \mathcal{E}$ in node $g \in \mathcal{G}$ of scenario tree

For all $i \in \mathcal{I}^N$, $g \in \mathcal{G} : t(e(g)) = 1, \cdots, T - S_{n(i)}$:

$$\begin{aligned} x_{i,g} &\in \{0, 1\} \\ x_{i,\beta(g)} &\leq x_{i,g} \end{aligned} \tag{5}$$

GEP. Forcing the upper bounding of the number of the plants / farms of candidate technology $n \in \mathcal{N}$ whose construction can be started at (first) period $t(e)$ of stage $e(g) \in \mathcal{E}$ in node $g \in \mathcal{G}$ of the scenario tree

$$\sum_{i \in \mathcal{I}^n} \sum_{g \in \mathcal{G}: t(e(g))=1,\cdots,T-S_{n(i)}} (x_{i,g} - x_{i,\beta(g)}) \leq \bar{I}^n \qquad (6)$$

GEP. Electricity generation lower and upper bounding for current power plant / farm i at period $t \in \mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$$\underline{P}_{i,t} \leq p_{i,t,g} \leq \overline{P}_{i,g} \qquad \forall i \in \mathcal{I}^K, \tag{7}$$

where

$$\overline{P}_{i,g} = \begin{cases} \frac{1}{1000} P_i H_{i,g} (1 - \nu_i) & \text{if} \quad t \leq L_i \\ 0 & \text{if} \quad t > L_i \end{cases} \tag{8}$$

Parameter $H_{i,g}$ takes into account possible plant / farm breakdown and maintenance.

GEP. Electricity generation lower and upper bounding for power plant / farm i of candidate technology n at period $t \in \mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree (c.)

The electricity generation is lower bounded by the *conditional* minimum $\underline{P}_{i,t}$.

$$\underline{P}_{i,t} x_{i,g'} \leq p_{i,t,g} \leq \overline{P}_{n(i),g} x_{i,g'} \qquad \forall i \in \mathcal{I}^N, \tag{9}$$

where $g' \in \mathcal{A}^g : t(e(g')) = t(e(g)) - S_{n(i)}$.
The maximum electricity generation $\overline{P}_{n,g}$ of a power plant / farm of technology $n \in \mathcal{N}$ is defined as

$$\overline{P}_{n,g} = \frac{1}{1000} P_n H_{n,g} (1 - \nu_n) \tag{10}$$

Note: Parameter $H_{n,g}$ takes into account possible plant / farm breakdown and maintenance.

GEP. Defining the bounding of thermal power generation, due to raw material availability at period $t \in \mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$$
\begin{aligned}
p_{i,t,g} &= \tfrac{1}{1000}\varphi_{ij} m_{ij,t,g} && \forall j \in \mathcal{J}_i,\ i \in \mathcal{I}^{K^T} \\
p_{i,t,g} &= \tfrac{1}{1000}\varphi_{n(i)j,g} m_{ij,t,g} && \forall j \in \mathcal{J}_i,\ i \in \mathcal{I}^{N^T} \\
e_{j,t,g} + n_{j,t,g} - \sum_{i \in \mathcal{I}^{K^T} \cup \mathcal{I}^{N^T}} m_{ij,t,g} &= e_{j,t+1,\hat{g}} && \forall j \in \mathcal{J},
\end{aligned}
\tag{11}
$$

where $\hat{g} = g$ for $t+1 \in \mathcal{T}^{e(g)}$ and otherwise, $\hat{g} = g'$ **for all** $g' \in \mathcal{G}^{e'}$ being $e' = e(g) + 1$ (notice that $t + 1 = t(e')$).

$$
n_{j,t,g} \leq \overline{N}_{j,g} \qquad \forall j \in \mathcal{J} \tag{12}
$$

GEP. Defining the 0-1 character of the x-variables for the new power turbine ij for $j \in \tilde{\mathcal{D}}_i$, $i \in \mathcal{I}^V$ that can start its construction at (first) period $t(e(g))$ of stage $e(g) \in \mathcal{E}$ in node $g \in \mathcal{G}$ of the scenario tree

For all ij for $j \in \tilde{\mathcal{D}}_i$, $i \in \mathcal{I}^V$, $g \in \mathcal{G} : t(e(g)) = 1, \cdots, T - S_{ij}$:

$$
\begin{aligned}
x_{ij,g} &\in \{0,1\} \\
x_{ij,\beta(g)} &\leq x_{ij,g}
\end{aligned}
\tag{13}
$$

GEP. Defining the replicated network of the reservoirs $i \in \mathcal{I}^V$ in the hydro valleys along the periods $\{t\}$ of the time horizon for period $t \in \mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$$w_{i,t,g} + \sum_{j \in \mathcal{U}_i} W_{i,g} = \sum_{j \in \mathcal{D}_i} r_{ij,t,g} + w_{i,t+1,\hat{g}}, \qquad (14)$$

where $\hat{g} = g$ for $t + 1 \in \mathcal{T}^{e(g)}$ and otherwise, $\hat{g} = g'$ **for all** $g' \in \mathcal{G}^{e'}$ being $e' = e(g) + 1$ (notice that $t + 1 = t(e')$).

$$\underline{W}_{i,g} \leq w_{i,t,g} \leq \overline{W}_{i,g}$$
$$\underline{R}_{ij,g} \leq r_{ij,t,g} \leq \overline{R}_{ij,g} \quad \forall j \in \mathcal{D}_i \qquad (15)$$

GEP. Electricity generation from current and potential new turbines in the reservoirs $i \in \mathcal{I}^V$ of the hydro valleys for period $t \in \mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$$p_{i,t,g} = \sum_{j \in \mathcal{D}_i \setminus \tilde{\mathcal{D}}_i} \frac{1}{1000} (\phi^K_{ij} \min\{r_{ij,t,g}, \overline{R}^K_{ij}\})(1 - x_{ij,g''}) + \sum_{j \in \tilde{\mathcal{D}}_i} \frac{1}{1000} (\phi^N_{ij,g} \min\{r_{ij,t,g}, \overline{R}^N_{ij,g}\}) x_{ij,g''} \quad (16)$$

where $g'' \in \mathcal{A}^g : t(e(g'')) = t(e(g)) - S_{ij}$ is the ancestor node in the scenario tree to node g, such the value 1 of variable $x_{ij,g''}$ defines that the new turbines in the reservoirs $i \in \mathcal{I}^V$ have started its construction **by** period $t(e(g))$ and, then, the entity is available for production.

GEP. Electricity served demand by the whole generation system $\mathcal{I}^K \cup \mathcal{I}^V \cup \mathcal{I}^N$ in period $t \in \mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$$\sum_{i \in \mathcal{I}^K \cup \mathcal{I}^V \cup \mathcal{I}^N} p_{i,t,g} = s_{t,g}$$

$$0 \leq s_{t,g} \leq \overline{D}_{t,g} \tag{17}$$

GEP. Periodic investment cost allocation of new power plants / farms for the set of periods $\mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree

$$\frac{1}{(1+\rho)^{t(e(g))}}\Big(\sum_{i\in\mathcal{I}^N} R_i x_{i,g'} + \sum_{i\in\mathcal{I}^V}\sum_{j\in\tilde{\mathcal{D}}_i} R_{ij} x_{ij,g''}\Big) \le B_g, \qquad (18)$$

where $g' \in \mathcal{A}^g : t(e(g')) = t(e(g)) - S_{n(i)}$ (res., $g'' \in \mathcal{A}^g : t(e(g'')) = t(e(g)) - S_{ij}$) is the ancestor node in the scenario tree to node g, such the value 1 of variable $x_{i,g'}$ (res., $x_{ij,g''}$) defines that the related entity has started its construction **by** period $t(e(g))$ and, then, the entity is available for production.

GEP. Periodic investment cost allocation of new power plants / farms for the set of periods $\mathcal{T}^{e(g)}$ in node $g \in \mathcal{G}$ of the scenario tree (c.)

Remember that R_i (res. R_{ij}) are the debt repayment per period for investment in power plant / farm i of candidate technology $n(i)$ (res. new turbine(s) for 'canal' ij of hydro power generation), such that

$$\begin{aligned} R_i &= \frac{1000 \cdot M_i \cdot P_{n(i)} \cdot \rho}{1-(\frac{1}{1+\rho})^{L_{n(i)}}} & i \in \mathcal{I}^N \\ R_{ij} &= \frac{1000 \cdot M_{ij} \cdot P_{ij} \cdot \rho}{1-(\frac{1}{1+\rho})^{L_{ij}}} & j \in \tilde{\mathcal{D}}_i,\ i \in \mathcal{I}^V \end{aligned} \qquad (19)$$

The amount of electricity $q_{t,g}$ for which, at period t in the node g of the scenario tree, the corresponding Green Certificates are bought if $q_{t,g} < 0$, or sold if $q_{t,g} > 0$ is defined as follows,

$$q_{t,g} = \sum_{i \in \mathcal{I}^V \cup \mathcal{I}^{N^R} \cup \mathcal{I}^{K^R}} p_{i,t,g} - \gamma_t \sum_{i \in \mathcal{I}} p_{i,t,g}$$

$$\overline{Q}_g \leq q_{t,g} \qquad (20)$$

The amount $o_{t,g}$ of CO_2 that can be emitted (and, then, paid for) is defined as follows,

$$o_{t,g} = \sum_{i \in \mathcal{I}^{K^T}} \zeta_i p_{i,t,g} + \sum_{i \in \mathcal{I}^{N^T}} \zeta_{n(i),g} p_{i,t,g}$$

$$0 \leq o_{t,g} \leq \mu_g \sum_{i \in \mathcal{I}^{N^T} \cup \mathcal{I}^{K^T}} p_{i,t,g} \qquad (21)$$

GEP. Warning. Linking variables from ancestor nodes $g' \in \mathcal{A}^g$ into constraints in node $g \in \mathcal{G}$ of the scenario tree

- Continuous (water stored) variable $w_{i,t+1,\hat{g}}$ and continuous (raw material stored) variable $e_{j,t+1,\hat{g}}$ through scheme

  $\hat{g} = g$ for $t+1 \in \mathcal{T}^{e(g)}$ and otherwise, $\hat{g} = g'$ for all $g' \in \mathcal{G}^{e'}$

  being $e' = e(g) + 1$.
- 0-1 variables for defining the generation capacity expansion:

  $x_{i,g}$ [−], for potential new power plant / farm $i \in \mathcal{I}^N$, for $g \in \mathcal{G} : t(e(g)) = 1, \cdots, T - S_{n(i)}$.

  $x_{ij,g}$ [−], for potential new turbine(s) in 'canal' ij for $j \in \tilde{\mathcal{D}}_i$, $i \in \mathcal{I}^V$, for $g \in \mathcal{G} : t(e(g)) = 1, \cdots, T - S_{ij}$.

Contents

- **ELECTRICITY TRANSMISSION NETWORK EXPANSION PLANNING (NEP): MULTISTAGE STOCHASTIC MIXED 0-1 MODELING**

NEP. European policy crossing borders: Examples

- Wind energy could be generated from the North Sea and South of Spain,
- Solar energy can be generated from South Spain and South Portugal,
- Biomass could be generated from neighbors to the European region. And
- all together can be used for satisfying electricity at multiple European points far away from the physical sources.

NEP.
Main uncertain parameters in the electricity system

- Electricity demand from the network nodes.
- Electricity offer from the power generation nodes.
- Electricity loss of new transmission technologies.
- Characteristics (i.e., maximum energy flow and reactance) of cable types on new energy transmission lines.
- Fixed and variables costs of energy transmission technologies.

- Cost of new transmission lines-
- Transmission Network reliability and resilence.
- EC directives on environmental issues and others.
- EU governments, etc.

NEP. Some European projects affecting Transmission systems

- ENTSO-e, European Network of Transmission System Operators for Electricity. Ten Year Network Development Plan 2010-2020. Those investments are very capital intensive (e.g., ENTSO-e members joint budget EURO 104 bn, 2012-2022) and long useful life (up to 40 years). See www.entsoe.eu.
- Desertec, project of a German consortium for installing RES power plants (over 20 GW, photovoltaic, solar power, wind, ...) in Sahara desert to be connected to European transmission system. See www.Desertec.org/fileadmin/downloads/desertec
- OWF: 5.3 GW installed in Europe (including London array, 1 GW). See EWEA. Wind in power. report 2012.
- See Achim Woyte and 3E OffshoreGrid. An Intelligent Energy Europe Project, 2009.
- London Array. See www.londonarray.com.
- NSCOGI. See www.The North Sea Countries' Offshore Grid Initiative.

Risk Neutral (RN). Parameters

- a_1^g, $b_1^{t,g}$, vectors of objective function coeffs for variables in vectors $x^g, y^{t,g}$, res.
- $h^{t,g}$, rhs for set of constraints related to node g in period t, for $t \in \mathcal{T}^{e(g)}$, $g \in \mathcal{G}$.
- $A_{t,g}^{g'}$, $B_{t,g}^{t',g'}$, matrices related to ancestor node g' and ancestor pair (t', g') in constraints related to pair (t, g), res., for $t' \in \mathcal{T}^{e(g')} : t' \le t, g' \in \mathcal{A}^g, t \in \mathcal{T}^{e(g)}, g \in \mathcal{G}$.

  Notice that $t' < t$ for $g' \neq g$.

NEP. Risk Neutral (RN). Vector of variables x^g in scenario tree node $g \in \mathcal{G}$

A vector of 0-1 variables for defining the new electricity transmission lines, such that the value 1 for each element means that its construction starts in the (first period of) stage $e(g)$ for node g and otherwise, 0. A related element can be:

- a cable type of a given new transmission line

NEP. Risk Neutral (RN). Vector of variables $y^{t,g}$ in period $t \in \mathcal{T}^{e(g)}$ of scenario tree node $g \in \mathcal{G}$

A vector of continuous variables, such that each element represents for period t in node g one of the following elements:

- electricity generated from power plants / farms,
- voltage angle at the energy transmission nodes,
- electricity energy flow through the transmission cables,
- served electricity demand from each node in the network, etc.

NEP. RN multistage mixed 0-1 DEM. Compact representation

Risk neutral model that synthesizes model (25)-(32) (see below):

$$z_{RN} = \max \sum_{g \in \mathcal{G}} w^g \left(a_1^g x^g + \sum_{t \in \mathcal{T}^{e(g)}} b_1^{t,g} y^{t,g} \right) \tag{22}$$

subject to

$$\sum_{g' \in \mathcal{A}^g} \sum_{t' \in \mathcal{T}^{e(g')} : t' \leq t} \left(A_{t,g}^{g'} x^{g'} + B_{t,g}^{t',g'} y^{t',g'} \right) = h^{t,g} \quad \forall t \in \mathcal{T}^{e(g)}, \; g \in \mathcal{G}$$

$$x^g \in \{0,1\}^{nx(g)}, \; y^{t,g} \in \mathbb{R}^{ny(t,g)} \quad \forall t \in \mathcal{T}^{e(g)}, \; g \in \mathcal{G}$$

Note: The constraint qualification $t' \leq t$ is only active for $g' = g$.

NEP. Constraints

For the periods along the time horizon over the scenarios, satisfying automatically the NAC (since the above representation of the model is a compact one): lower / upper bounds on:

- First Kirchhoff law balancing power in the nodes of current transmission lines and Second Kirchhoff law (voltage law) defining energy flow in the transmission network.
- First and Second Kirchhoff laws for the candidate technologies that are available once their construction is over.
- Periodic repayment of investment cost allocation of the cables of the new transmission lines, etc.

NEP. Modeling

- Approximation: A huge but very well structured multicriteria multistage stochastic mixed 0-1 linear optimization problem
- Key energy stakeholders (transmission system operators (TSO)) want to determine their optimal planning for investment in electricity transmission in a long term horizon and, on the other hand, environmental entities want to get their goals.

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NEP. Problem description

- TSO's aim: Minimizing expected cost along the time horizon at NPV subject to feasible constraints for nodes (i.e., scenario groups) in the scenario tree.
- Variable and fixed costs also greatly differ among the line transmission technologies.
- Investment costs on cables for energy transmission lines depend on transmission types.
- Uncertainty on electricity offer and demand as well as on the transmission line disruption along the time horizon.

- A risk is associated with the expected cost of the electricity transmission network expansion, due to the uncertainty on the main parameters.
- Our risk management proposal: A mixture of time-inconsistent and time-consistent measures, such as Conditional Value-at-Risk (TCVaR / ECVaR) or stochastic dominance (TSD/ESD) measures.

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NEP. Problem description (c.)

- The proposed model determines the evolution of the electricity transmission network along the time horizon. So, it determines for every transmission line technology:
  - **site location of transmission line**
  - **year to start the construction**

  depending on the node in the scenario tree (i.e., scenario group) along the time horizon.
- It could be possible that at the year when the new transmission line is ready for being in operation, **the realization of uncertain electricity offer and demand and other key uncertain elements be drastically changed along the scenario tree from the node in tree where the construction have started.**

NEP. SMILP approximating model due to its SMINLP complexity and long time horizon

- Linear functions approximate nonlinearities on electricity flow and flow losses, etc.
- Some peculiarities of the transmission system are relaxed. They are related to substations, cables, transformers and converters types and their features, peculiarities of Off-shore Wind Farms, sets of AC and DC cables, correspondence between transformer or converter types and their respective voltage levels, etc.

NEP. Problem modeling. Network sets

- $\mathcal{I}^G$, Power generation nodes.
- $\mathcal{I}^T$, Pure energy transmission nodes (i.e., they are not power generation plants / farms).
- $\mathcal{I}$, Nodes in the transmission (connected) network, such that $\mathcal{I} = \mathcal{I}^G \cup \mathcal{I}^T$. Note: It is assumed that any node in the transmission network can have electricity demand (i.e., it could be zero).
- $\mathcal{L}^N$, Candidate (new) transmission lines to be installed in the network.
- $\mathcal{L}^K$, Current lines in the transmission network.
- $\mathcal{L}$, Transmission lines in the network (i.e., $\mathcal{L} = \mathcal{L}^K \cup \mathcal{L}^N$).
- $\mathcal{C}_{ij}$, Cables in transmission line $ij \in \mathcal{L}$.

In the electricity network:

- S_{ijc} $[t]$, Construction time periods required for cable $c \in \mathcal{C}_{ij}$ of new transmission line ij, for $ij \in \mathcal{L}^N$.
- L_{ijc} $[t]$, Industrial life of cable $c \in \mathcal{C}_{ij}$ of current transmission line ij, for $ij \in \mathcal{L}^K$.

ρ $[-]$, Discount rate.

M_{ijc}, R_{ijc} [*MEURO*], Investment cost and its periodic allocation (see below), res., required by cable $c \in \mathcal{C}_{ij}$ of new line $ij \in \mathcal{L}^N$ in the transmission network.

NEP. Problem modeling. Uncertain parameters for period t in $\mathcal{T}^{e(g)}$ in scenario tree node $g \in \mathcal{G}$

$P_{i,t,g}$, , electricity to generate in generation node $i \in \mathcal{I}^G$.

$\overline{D}_{i,t,g}$ $[GWh]$, electricity demand in network node $i \in \mathcal{I}$.

NEP problem modeling. Uncertain parameters for any period from set $\mathcal{T}^{e(g)}$ in scenario tree node $g \in \mathcal{G}$

For each network node $i \in \mathcal{I}$:

$\overline{V}_{i,g}$ [rad], Maximum voltage angle allowed.

For each cable $c \in \mathcal{C}_{ij}$ of new transmission line ij, so, $ij \in \mathcal{L}^N$;

$\overline{F}_{ijc.g}$ [GWh], Maximum flow allowed.

$\lambda_{ijc,g}$ [oh], Reactance.

$\sigma_{ijc,g}$ [$-$], Loss rate.

NEP. Problem modeling. Uncertain fixed cost parameters for any period from set $\mathcal{T}^{e(g)}$ in scenario tree node $g \in \mathcal{G}$

$c_{ijc,g}$ [$kEURO$], cable $c \in \mathcal{C}_{ij}$ of new transmission line $ij \in \mathcal{L}^N$ in the electricity network.

NEP. Problem modeling. Uncertain economic parameters for whole set of periods in $\mathcal{T}^{e(g)}$ in scenario tree node $g \in \mathcal{G}$

B_g [*MEURO*], Upper bound on the total investment cost amortization that is allocated, due to the availability of the new transmission lines.

NEP. Problem modeling. 0-1 x-variables for new transmission lines in the electricity network at (first) period $t(e(g))$ of stage $e(g) \in \mathcal{E}$ in scenario tree node $g \in \mathcal{G}$

$x_{ijc,g}$ $[-]$, its value 1 means that the construction of the cable $c \in \mathcal{C}_{ij}$ of new transmission line $ij \in \mathcal{L}^N$ starts **by** that period and otherwise, 0, for $g \in \mathcal{G} : t(e(g)) = 1, \cdots, T - S_{ijc}$.

Observe that, without loss of generality, it is assumed that the starting construction period can only be performed at the first period of any stage along the time horizon.

NEP. Problem modeling. Decision continuous variables for time period $t \in \mathcal{T}^{e(g)}$ of stage $e(g) \in \mathcal{E}$ in scenario tree node $g \in \mathcal{G}$

For the electricity network:

- $v_{i,t,g}$ [rad], Voltage angle at node $i \in \mathcal{I}$.
- $f_{ijc,t,g}$ [GWh], Energy flow through cable $c \in \mathcal{C}_{ij}$ of line $(ij) \in \mathcal{L}$.
- $s_{i,t,g}$ [GWh], Served electricity demand from node $i \in \mathcal{I}$.

Minimizing the NPV of the expected cost of the new transmission lines plus the fixed cost of the cables of the current transmission lines:

$$z = \min \sum_{\omega \in \Omega} w^{\omega} z^{\omega} + \sum_{ij \in \mathcal{L}^K} \sum_{c \in \mathcal{C}_{ij}} c_{ijc} \sum_{g \in \mathcal{G}} w^g \frac{1}{(1+\rho)^{t(e(g))}}, \tag{23}$$

where z^{ω} gives the NPV of cost (24) under scenario $\omega \in \Omega$, such that min z (23) is subject to constraints (25)-(32), and $t \equiv t(e(g))$.

NEP. Problem modeling.
Risk Neutral objective function (c.)

Elements of NPV of new lines transmission cost z^ω under scenario $\omega \in \Omega$:

- Periodic debt repayment of all new transmission lines
- Fixed transmission cost of available new transmission lines.

z^ω function

$$z^\omega = \sum_{g\in\mathcal{A}^\omega} \frac{1}{(1+\rho)^{t(e(g))}} \sum_{ij\in\mathcal{L}^N} \sum_{c\in\mathcal{C}_{ij}} (R_{ijc} + c_{ijc,g}) x_{ijc,g'} \qquad (24)$$

where $g' \in \mathcal{A}^g : t(e(g')) = t(e(g)) - S_{ijc}$ is the ancestor node in the scenario tree to node g, such the value 1 of variable $x_{ijc,g'}$ defines that the cable $c \in \mathcal{C}_{ij}$ of new transmission line $ij \in \mathcal{L}^N$ has started its construction **by** period $t(e(g))$ and, then, it is available for production.

NEP.

Defining the 0-1 character of the x-step variables for the cable $c \in \mathcal{C}_{ij}$ of new transmission line $ij \in \mathcal{L}^N$ that can started its construction **by** the (first) period $t(e)$ of stage $e(g) \in \mathcal{E}$ in scenario tree node $g \in \mathcal{G}$

For all $i \in \mathcal{I}^N, g \in \mathcal{G} : t(e(g)) = 1, \cdots, T - S_{ijc}$:

$$x_{ijc,g} \in \{0,1\}$$
$$x_{ijc,\beta(g)} \leq x_{ijc,g} \qquad (25)$$

NEP. First Kirchhoff law balancing power in the nodes of current transmission lines for period $t \in \mathcal{T}^{e(g)}$ in scenario tree node $g \in \mathcal{G}$

For energy generation nodes $i \in \mathcal{I}^K \cup \mathcal{I}^N \cup \mathcal{I}^V$:

$$\sum_{j:(j,i)\in\mathcal{L}} \sum_{c\in\mathcal{C}^{ji}} \sigma_{jic,g} f_{jic,t,g} + P_{i,t,g} - \sum_{j:(i,j)\in\mathcal{L}} \sum_{c\in\mathcal{C}_{ij}} \sigma_{ijc,g} f_{ijc,t,g} + s_{i,t,g} = \overline{D}_{i,t,g} \tag{26}$$

For energy transmission nodes $i \in \mathcal{I}^T$:

$$\sum_{j:(j,i)\in\mathcal{L}} \sigma_{jic,g} f_{jic,t,g} - \sum_{j:(i,j)\in\mathcal{L}} \sum_{c\in\mathcal{C}_{ij}} \sigma_{ijc,g} f_{ijc,t,g} + s_{i,t,g} = \overline{D}_{i,t,g} \tag{27}$$

Note: For both types of nodes, $\sigma_{ijc,g} \equiv \sigma_{ijc}$ for $ij \in \mathcal{L}^K$.

NEP. Second Kirchhoff law (Voltage law) defining energy flow in the transmission network for period $t \in \mathcal{T}^{e(g)}$ in scenario tree node $g \in \mathcal{G}$

For current energy transmission network $c \in \mathcal{C}_{ij}$, $ij \in \mathcal{L}^K$:

$$f_{ijc,t,g} = \frac{v_{i,t,g} - v_{j,t,g}}{\lambda_{ijc}} \tag{28}$$

For expansion energy transmission network $c \in \mathcal{C}_{ij}$, $ij \in \mathcal{L}^N$:

$$-M(1 - x_{ijc,g'}) \leq f_{ijc,t,g} - \frac{v_{i,t,g} - v_{j,t,g}}{\lambda_{ijc,g}} \leq M(1 - x_{ijc,g'}), \tag{29}$$

where $g' \in \mathcal{A}^g : t(e(g')) = t(e(g)) - S_{ijc}$ is the ancestor node in the scenario tree to node g, such the value 1 of variable $x_{ijc,g'}$ defines that the cable $c \in \mathcal{C}_{ij}$ of the new transmission line $ij \in \mathcal{L}^N$ has started its construction **by** period $t(e(g))$ and, then, it is available for operation.

NEP. Periodic investment cost allocation of cables of new transmission lines in the electricity network for the set of periods $\mathcal{T}^{e(g)}$ in scenario tree node $g \in \mathcal{G}$

$$\frac{1}{(1+\rho)^{t(e(g))}} \sum_{ij \in (\mathcal{L}^N)} \sum_{c \in \mathcal{C}_{ij}} R_{ijc} x_{ijc,g'} \leq B_g \tag{30}$$

R_{ijc} is the debt repayment per period for investment in cable $c \in \mathcal{C}_{ij}$ of transmission line ij, such that

$$R_{ijc} = \frac{1000 \cdot M_{ijc} \cdot \rho}{1 - (\frac{1}{1+\rho})^{L_{ijc}}} \tag{31}$$

NEP. Variables defining for period $t \in \mathcal{T}^{e(g)}$ in scenario tree node $g \in \mathcal{G}$

$$
\begin{aligned}
& s_{i,t,g} \in \mathbb{R}^{+} && \forall i \in \mathcal{I} \\
& 0 \leq v_{i,t,g} \leq \overline{V}_{i,g} && \forall i \in \mathcal{I} \\
& -\overline{F}_{ijc,g} \leq f_{ijc,t,g} \leq \overline{F}_{ijc,g} && \forall c \in \mathcal{C}_{ij},\ ij \in \mathcal{L}^{K} \\
& -\overline{F}_{ijc.g} x_{ijc,g'} \leq f_{ijc,t,g} \leq \overline{F}_{ijc.g} x_{ijc,g'} && \forall c \in \mathcal{C}_{ij},\ ij \in \mathcal{L}^{N},
\end{aligned}
\tag{32}
$$

where $g' \in \mathcal{A}^{g} : t(e(g')) = t(e(g)) - S_{ijc}$ is the ancestor node as above.

Contents

- **TIME-INCONSISTENT STOCHASTIC DOMINANCE (TSD) RISK AVERSE MEASURE**

Scenario tree notation

$\mathcal{E}$, set of the stages along the horizon.

Ω, set of scenarios.

$\mathcal{G}$, set of nodes in the tree.

$\Omega^g \subseteq \Omega$, set of scenarios in a group with a 1-1 correspondence to node g, for $g \in \mathcal{G}$.

$\mathcal{G}^e \subset \mathcal{G}$, set of nodes for stage e, for $t \in \mathcal{T}$.

$e(g)$, stage to which node g belongs to, for $g \in \mathcal{G}$.

w^ω, weight or probability assigned to scenario $\omega \in \Omega$.

w^g, weight or probability assigned to node $g \in \mathcal{G}$. It is computed as $w^g = \sum_{\omega \in \Omega} w^\omega$

$\mathcal{F}$, set of functions / criteria to be satisfied, such that function indexed by $f = 1$ is the objective fun to maximize.

Scenario tree notation (c.)

$\tilde{A}^g$, set of ancestor nodes to node g (including itself), for $g \in \mathcal{G}$.

$\mathcal{A}^g \subseteq \tilde{A}^g$, set of ancestor nodes to node g in the scenario tree (including itself) with nonzero elements in constraints of node g, for $g \in \mathcal{G}$.

$\beta(g)$, immediate ancestor node in the scenario tree of node g, for $g \in \mathcal{G}$. Let us assume that $\beta(1)$ is empty.

$\mathcal{S}^g$, set of successor nodes in the scenario tree to node g (excluding itself) (i.e., nodes in the subtree whose root is node g), for $g \in \mathcal{G}$,

Note: Set of scenarios Ω^g of any leaf node g (i.e., last node) is singleton. Let us assume that $\omega = g$ for $\omega \in \Omega^g$ and $g \in \mathcal{G}^{|\mathcal{E}|}$.

Scenario tree notation (c.)

$\mathcal{T}$, set of periods (usually, years) in the time horizon. Last period $T = |\mathcal{T}|$.

$\mathcal{T}^e$, set of (consecutive) periods in stage e, for $e \in \mathcal{E}$, $\mathcal{T}^e \cap \mathcal{T}^{e'} = \emptyset$, $e, e' \in \mathcal{E} : e \neq e'$, $\mathcal{T} = \cup_{e \in \mathcal{E}} \mathcal{T}^e$.

$t(e)$, first period in set $\mathcal{T}^e$, for $e \in \mathcal{E}$.

Let (t, g) denote the pair of indexes for period t and node g, for $t \in \mathcal{T}^{e(g)}, g \in \mathcal{G}$.

Example of four-stage scenario tree (15 nodes, 8 scenarios)

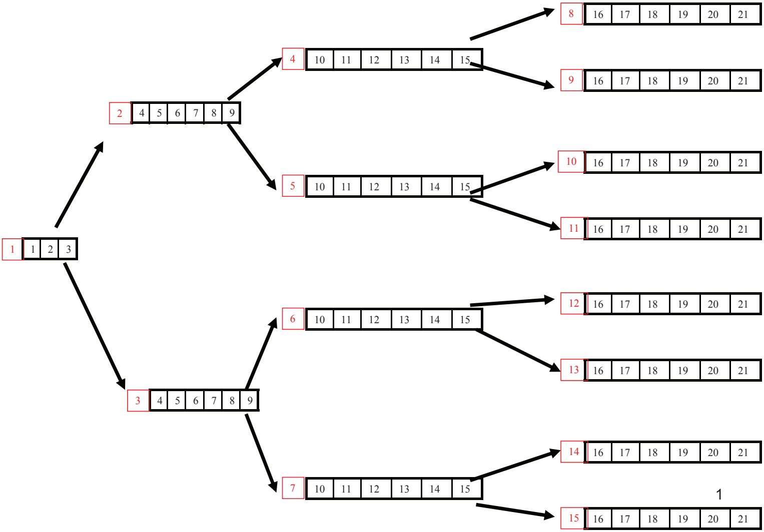

Figura: Vespucci CMS'11

Risk Neutral (RN). Parameters

- a_1^g, $b_1^{t,g}$, vectors of objective function coeffs for variables in vectors $x^g, y^{t,g}$, res.
- $h^{t,g}$, rhs for set of constraints related to node g in period t, for $t \in \mathcal{T}^{e(g)}$, $g \in \mathcal{G}$.
- $A_{t,g}^{g'}$, $B_{t,g}^{t',g'}$, matrices related to ancestor node g' and ancestor pair (t', g') in constraints related to pair (t, g), res., for $t' \in \mathcal{T}^{e(g')} : t' \leq t, g' \in \mathcal{A}^g, t \in \mathcal{T}^{e(g)}, g \in \mathcal{G}$.

  Notice that $t' < t$ for $g' \neq g$.

Risk Neutral (RN). Vector of variables x^g in scenario tree node $g \in \mathcal{G}$

A vector of 0-1 variables for defining the electricity capacity expansion, such that the value 1 for each element means that its construction starts in the (first period of) stage $e(g)$ for node g and otherwise, 0. A related element can be:

- For power generation capacity expansion planning:
  - generation plant, wind farm, solar farm, photovoltaic farm of a given candidate technology,
  - hydro turbine
- For new transmission lines planning problem:
  - cable in a given new transmission line

Risk Neutral (RN). Vector of variables $y^{t,g}$ in period $t \in \mathcal{T}^{e(g)}$ of scenario tree node $g \in \mathcal{G}$

A vector of continuous variables, such that each element represents for period t in node g one of the following elements:

- For power generation capacity expansion planning:
    - Electricity generated from each available current power plant / farm and those of the candidate technologies that being previously started their construction are already available,
    - Raw material availability for the thermal plants,
    - Replicated network definition of the reservoirs for the hyper-hydro valleys,
    - Electricity generated from each available current and potential new turbines, if they are available,
    - Periodic repayment of investment cost allocation of new power plants / farms,
    - Green Certificates and CO_2 emission definition,

Risk Neutral (RN). Vector of variables $y^{t,g}$ in period $t \in \mathcal{T}^{e(g)}$ of scenario tree node $g \in \mathcal{G}$ (c.)

A vector of continuous variables, such that each element represents for period t in node g one of the following elements (c.):

- For new transmission lines planning problem:
  - electricity generated by power plants / farms,
  - voltage angle at the energy transmission nodes,
  - electricity energy flow through the transmission cables,
  - served electricity demand from each node in the network, etc.

RN multistage mixed 0-1 DEM. Compact representation

Risk neutral model that synthesizes each model (5)-(21) and (25)-(32):

$$z_{RN} = \max \sum_{g \in \mathcal{G}} w^g \left(a_1^g x^g + \sum_{t \in \mathcal{T}^{e(g)}} b_1^{t,g} y^{t,g} \right) \tag{33}$$

subject to

$$\sum_{g' \in \mathcal{A}^g} \sum_{t' \in \mathcal{T}^{e(g')} : t' \leq t} \left(A_{t,g}^{g'} x^{g'} + B_{t,g}^{t',g'} y^{t',g'} \right) = h^{t,g} \quad \forall t \in \mathcal{T}^{e(g)},\ g \in \mathcal{G}$$

$$x^g \in \{0,1\}^{nx(g)},\ y^{t,g} \in \mathbb{R}^{ny(t,g)} \quad \forall t \in \mathcal{T}^{e(g)},\ g \in \mathcal{G}$$

Note: The constraint qualification $t' \leq t$ is only active for $g' = g$.

TSD/ESD risk averse measures. Motivation

- The risk neutral (RN) model maximizes the expected profit over the scenarios along the time horizon.
- However, it ignores the variability of the objective function value over the scenarios, in particular the “left” tail (for maximization) of the non-wanted scenarios and, additionally,

  it only considers the principal function / criterion.
- There are some risk averse approaches that additionally deal with risk management; among them, TSD/ESD reduce the risk of the negative impact of the solutions in non-wanted scenarios in a better way than the others under some circumstances.

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Time-inconsistent TSD risk averse measure. Motivation (c)

- The measure also aims to maximize the objective function expected value as RN.
- Additionally, a modeler-driven set of given thresholds on the value of given functions / criteria for each node in selected stages in the scenario tree should be satisfied with a bound *target* on the deficit (shortfall) on reaching each threshold, a bound *target* on the probability of having deficit and a bound *target* on the expected deficit.
- TSD risk averse TSD measure (Alonso-Ayuso et al., EJOR'14; LFE-Garín-Merino-Pérez, 2015) for multistage mixed 0-1 programs at the price of including some new variables and constraints, as a mixture of the FSD:first- (Gollmer-Neise-Schultz SIOPT'08), and SSD:second order- (Gollmer-Gotzes-Schultz MP'11) stochastic dominance constraints induced by integer-linear recourse for two-stage

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Multicriteria in GEP / NEP

There is not a unique function / criterion to consider. Rather, it is a multicriteria problem.

- Maximizing NPV of expected investment and consumer stakeholders goals over the scenarios along the time horizon subject to risk reduction of the negative impact of non-wanted scenarios on **multiple types of utility objectives and stakeholders**:
  - Maximizing power share of cleaner, safer and efficient -cheaper- energy accessible to all consumption nodes.
  - Minimizing cost investment from private and public institutions,
  - Generation and Transmission Network reliability.
  - EC directives on environmental issues and others.
  - EU governments, etc.

Time-inconsistent TSD risk averse measure: Set of modeler-driven functions and profiles

- $\mathcal{F}$, set of functions / criteria to be satisfied, such that function $f = 1$ is the objective fun to maximize.
- $\mathcal{E}_f \subseteq \mathcal{E}$, set of stages where **Time Stochastic Dominance (TSD)** has to be considered for function / criterion $f \in \mathcal{F}$.
- Set of profiles, say $\mathcal{P}_f^e$ for $e \in \mathcal{E}_f$, $f \in \mathcal{F}$,
- For each profile $p \in \mathcal{P}_f^e$ in TSD stage $e \in \mathcal{E}_f$ for function / criterion $f \in \mathcal{F}$:
  - ϕ^p, function f threshold to be satisfied up to the last period in any node g of stage e in the scenario tree, for $g \in \mathcal{G}^e$.
  - D^p, upper bound *target* on the deficit (shortfall) that is allowed on reaching threshold ϕ^p up to the last period in any node g of stage e, for $g \in \mathcal{G}^e$.
  - $\overline{d}^p$, upper bound *target* on the expected deficit on reaching threshold ϕ^p.
  - $\overline{\nu}^p$, upper bound *target* on the fraction of nodes with deficit on reaching threshold ϕ^p.

Time-inconsistent TSD risk averse measure: Additional variables

For each pair node (g, p), where g is the scenario tree and p is the profile in TSD stage e and function / criterion f, for $g \in \mathcal{G}^e, p \in \mathcal{P}^e_f, e \in \mathcal{E}_f, f \in \mathcal{F}$:

- $d^{g,p}$, deficit (shortfall) continuous variable that, obviously, is equal to the difference (if it is positive) between threshold ϕ^p and the value of function / criterion f up to last period of node g.
- $\nu^{g,p}$, 0-1 variable such that its value is 1 if $d^{g,p} > 0$ and otherwise, 0.

Time-inconsistent TSD risk averse measure: Model

$$z_{TSD} = \max \sum_{g \in \mathcal{G}} w^g \left(a_1^g x^g + \sum_{t \in \mathcal{T}^{e(g)}} b_1^{t,g} y^{t,g} \right)$$
$$- \sum_{f \in \mathcal{F}} \sum_{e \in \mathcal{E}_f} \sum_{p \in \mathcal{P}_f^e} \left(M_D^p \varepsilon_D^p + M_{\underline{d}}^p \varepsilon_{\underline{d}}^p + M_{\underline{\nu}}^p \varepsilon_{\underline{\nu}}^p \right)$$

subject to

$$\sum_{g' \in \mathcal{A}^g} \sum_{t' \in \mathcal{T}^{e(g')}: t' \leq t} \left(A_{t,g}^{g'} x^{g'} + B_{t,g}^{t',g'} y^{t',g'} \right) = h^{t,g} \quad \forall t \in \mathcal{T}^{e(g)}, \, g \in \mathcal{G}$$
$$x^g \in \{0,1\}^{nx(g)}, \; y^{t,g} \in \mathbb{R}^{ny(t,g)} \quad \forall t \in \mathcal{T}^{e(g)}, \, g \in \mathcal{G}$$

and

Time-inconsistent TSD risk averse measure: Model (c.)

$$\sum_{g' \in \mathcal{A}^g} \left(a_f^{g'} x^{g'} + \sum_{t' \in \mathcal{T}^{e(g')}} b_f^{t',g'} y^{t',g'}\right) + d^{g,p} \geq \phi^p$$

$$\forall g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$d^{g,p} \leq D^p \nu^{g,p} + \varepsilon_D^p,\ \nu^{g,p} \in \{0,1\} \quad \forall g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$\sum_{g \in \mathcal{G}^e} w^g d^{g,p} \leq \overline{d}^p + \varepsilon_{\overline{d}}^p \quad \forall p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$\sum_{g \in \mathcal{G}^e} w^g \nu^{g,p} \leq \overline{\nu}^p + \varepsilon_{\overline{\nu}}^p \quad \forall p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}.$$

Time-inconsistent TSD risk averse measure: Model (c.)

$$d^{g,p} \in \mathbb{R}_+ \qquad \forall g \in \mathcal{G}^e, p \in \mathcal{P}^e_f, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$\varepsilon^p_D,\ \varepsilon^p_{\underline{d}},\ \varepsilon^p_{\underline{\nu}} \in \mathbb{R}_+ \quad \forall p \in \mathcal{P}^e_f, e \in \mathcal{E}_f, f \in \mathcal{F}, \qquad (34)$$

where

- ε^p_D, $\varepsilon^p_{\underline{d}}$ and $\varepsilon^p_{\overline{\nu}}$, slack variables for the violation of the targets D^p, $\overline{d}^p$ and $\overline{\nu}^p$, respectively, and
- M^p_D, $M^p_{\underline{d}}$ and $M^p_{\overline{\nu}}$ are the related penalization parameters for the slack variables.
- Notice that the appropriate hierarchy on the values of those parameters for the different functions / criteria help the decision maker (either GenCo or TSO) and concerned stakeholders to play with the potential enforcing of the TSD targets, depending on the priority-based classification of the functions / criteria.

Time-inconsistent TSD risk averse measure. Comments

- Following the rationale in Plugh'00 for the CVaR measure, it can be shown that TSD is a coherent risk measure, according to the standards setup in Artzner et al, MF'99 and ANOR'07, since it satisfies the properties: translation invariance, positive homogeneity, monotonicity and convexity.
- Additionally, it can also be shown that the time-consistency property (see the definition in Homem-de-mello & Pagnoncelli, EJOR'16 and below) of TSD measure depends on the bounds $D^p + \varepsilon^p_D$, $\overline{d}^p + \varepsilon^p_{\overline{d}}$ and $\overline{\nu}^p + \varepsilon^p_{\overline{\nu}}$, such the tighter they are, the lower the probability of TSD to be time-consistent. So, by extension, TSD is in general a time-inconsistent measure.

Contents

- **TIME-CONSISTENT EXPECTED STOCHASTIC DOMINANCE (ESD) RISK AVERSE MEASURE**

Time-consistent ESD measure. New parameters

- Let us consider the risk reduction by using the expected stochastic dominance (for short, ESD) measure.
- Remember $\mathcal{P}_f^e$ denote the set of profiles, for $e \in \mathcal{E}_f$, $f \in \mathcal{F}$.
- Each profile is included by the 4-tuple $(\phi^p, D^p, \overline{d}^p, \overline{\nu}^p)$:
  - ϕ^p, threshold to be satisfied by **any scenario in group** Ω^g for $g \in \mathcal{G}^e$, $e \in \mathcal{E}_f$, $f \in \mathcal{F}$,
  - D^p, upper bound *target* on the deficit (shortfall) allowed for any of those those scenarios,
  - $\overline{d}^p$, upper bound *target* on the expected deficit on reaching threshold ϕ^p that is allowed for those scenarios,
  - $\overline{\nu}^p$, upper bound of the failure probability on reaching the threshold.

$$z_{ESD} = \max \sum_{g\in\mathcal{G}} w^g \left(a_1^g x^g + \sum_{t\in\mathcal{T}^{e(g)}} b_1^{t,g} y^{t,g}\right) - \sum_{f\in\mathcal{F}} \sum_{e\in\mathcal{E}_f} \sum_{p\in\mathcal{P}_f^e} \left(M_D^p \varepsilon_D^p + M_{\underline{d}}^p \varepsilon_{\underline{d}}^p + M_{\underline{\nu}}^p \varepsilon_{\underline{\nu}}^p\right) \quad (35)$$

subject to

$$\sum_{g'\in\mathcal{A}^g} \sum_{t'\in\mathcal{T}^{e(g')}:t'\leq t} \left(A_{t,g}^{g'} x^{g'} + B_{t,g}^{t',g'} y^{t',g'}\right) = h^{t,g} \quad \forall t \in \mathcal{T}^{e(g)}, g \in \mathcal{G}$$

$$x^g \in \{0,1\}^{nx(g)}, \; y^{t,g} \in \mathbb{R}^{ny(t,g)} \quad \forall t \in \mathcal{T}^{e(g)}, g \in \mathcal{G}$$

and

Time-consistent ESD risk averse measure: Model (c.)

$$\sum_{g' \in \mathcal{A}^\omega} \left(a_f^{g'} x^{g'} + \sum_{t' \in \mathcal{T}^{e(g')}} b_f^{t',g'} y^{t',g'}\right) + d^{\omega,p} \geq \phi^p$$

$$\forall \omega \in \Omega^g, g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$d^{\omega,p} \leq D^p \nu^{\omega,p} + \varepsilon_D^p, \; \nu^{\omega,p} \in \{0,1\}$$

$$\forall \omega \in \Omega^g, g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$\sum_{\omega \in \Omega^g} w^\omega d^{\omega,p} \leq \overline{d}^p + \varepsilon_{\overline{d}}^p \quad \forall g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$\sum_{\omega \in \Omega^g} w^\omega \nu^{\omega,p} \leq \overline{\nu}^p + \varepsilon_{\overline{\nu}}^p \quad \forall g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$d^{\omega,p} \in \mathbb{R}_+ \quad \forall \omega \in \Omega^g, g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$$

$$\varepsilon_D^p,\ \varepsilon_{\overline{d}}^p,\ \varepsilon_{\overline{\nu}}^p \in \mathbb{R}_+ \quad \forall p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F} \tag{36}$$

Time-consistency property of ESD measure

- The ESD measure can be included in the family so-named *expected conditional risk measures* (ECRMs) considered in Homem-de-Mello & Pagnoncelli, EJOR'16, where the time-consistency property of those measures is proved, according to the definition introduced there.
- Notice that the proof only requires that the measure has the properties of translation-invariance and monotonicity. See some variants in Pflug & Pichler, MORS'15 and EJOR'16; Rudloff, Street & Valladao, EJOR'14; Ruszczyski, MPS'10, Shapiro, ORL'09, among others.
- In our context that definition is presented next.

- Let $\hat{x}^q$ and $\hat{y}^q$ $\forall q \in \mathcal{G}$ denote the vectors of the values of the variables in the vectors x^q and y^q of model *ESD*, res., for $q \in \mathcal{G}$, for any of the optimal solutions.
- Let $\tilde{z}_{ESD^g}$ denote the value of the sum of the terms in the objective function of model *ESD* related to the optimal values $\hat{x}^q$ and $\hat{y}^q$ of the variables in the vectors x^q and y^q, res., for $q \in \tilde{\mathcal{A}}^g \cup \mathcal{S}^g$ for any node $g \in \mathcal{G}$, i.e.,

$$\tilde{z}_{ESD^g} = \sum_{q \in \tilde{\mathcal{A}}^g \cup \mathcal{S}^g} w^q \big(a_1^q \hat{x}^q + \sum_{t \in \mathcal{T}^{e(q)}} b_1^{t,q} \hat{y}^{t,q} \big).$$

Time-consistency property of ESD measure (c.)

Let us consider the submodel ESD^g of model ESD related to any node g, for $g \in \mathcal{G}$, whose elements are as follows:

- The subtree that supports the submodel is given by the node set $\tilde{\mathcal{A}}^g \cup \mathcal{S}^g$, such that it is included by the nodes from the original scenario tree up to node g plus the subtree rooted in that node up to the leaves of the original scenario tree.
- The input data of the submodel is taken from the appropriate nodes in model ESD for any scenario tree of any set Ω.
- The variables in the vectors x^g and y^q $\forall q \in \mathcal{A}^g$ are fixed in submodel ESD^g to the related values in the vectors $\hat{x}^q$ and $\hat{y}^q$ in the optimal solution of model ESD that is being considered.

Time-consistency property of ESD measure. Submodel ESD^g, $g \in \mathcal{G}$

$$z_{ESD^g} = \text{máx} \sum_{q \in \tilde{\mathcal{A}}^g \cup \mathcal{S}^g} w^q \big(a_1^q x^q + \sum_{t \in \mathcal{T}^{e(q)}} b_1^{t,q} y^{t,q}\big)$$

$$-\sum_{f \in \mathcal{F}} \sum_{e \in \mathcal{E}_f : e > e(g)} \sum_{p \in \mathcal{P}_f^e} (M_D^p \varepsilon_D^p + M_{\underline{d}}^p \varepsilon_{\underline{d}}^p + M_{\underline{\nu}}^p \varepsilon_{\underline{\nu}}^p)$$

subject to

$$\sum_{q \in \mathcal{A}^{g'}} \sum_{t' \in \mathcal{T}^{e(q)} : t' \le t} (A_{t,g'}^q x^q + B_{t,g'}^{t',q} y^{t',q}) = h^{t,g'} \quad \forall t \in \mathcal{T}^{e(g')}, g' \in \mathcal{S}_g$$

$$x^q = \hat{x}^q,\ y^{t,q} = \hat{y}^{t,q} \quad \forall t \in \mathcal{T}^{e(q)}, q \in \tilde{\mathcal{A}}$$

$$x^q \in \{0,1\}^{nx(q)},\ y^{t,q} \in \mathbb{R}^{ny(t,q)} \quad \forall q \in \tilde{\mathcal{A}}^g \cup \mathcal{S}^g$$

and

Time-consistency property of ESD measure. Submodel ESD^g, $g \in \mathcal{G}$ (c.)

$$\sum_{q \in \mathcal{A}^\omega} (a_f^q x^q + \sum_{t \in \mathcal{T}^{e(q)}} b_f^q y^q) + d^{\omega,p} \geq \phi^p$$

$$\forall \omega \in \Omega^q, q \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}, e > e(g)$$

$$d^{\omega,p} \leq D^p \nu^{\omega,p} + \varepsilon_D^p, \; \nu^{\omega,p} \in \{0,1\}$$

$$\forall \omega \in \Omega^q, q \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}, e > e(g)$$

$$\sum_{\omega \in \Omega^q} w^\omega d^{\omega,p} \leq \overline{d}^p + \varepsilon_{\overline{d}}^p \quad \forall q \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}, e > e(g)$$

$$\sum_{\omega \in \Omega^q} w^\omega \nu^{\omega,p} \leq \overline{\nu}^p + \varepsilon_{\overline{\nu}}^p \quad \forall g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}, e > e(g)$$

Time-consistency property of ESD measure. Submodel ESD^g, $g \in \mathcal{G}$ (c.)

$$d^{\omega,p} \in \mathbb{R}_+ \qquad \forall \omega \in \Omega^q, q \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}, e > e(g)$$

$$\varepsilon_D^p,\ \varepsilon_{\overline{d}}^p,\ \varepsilon_{\overline{\nu}}^p \in \mathbb{R}_+ \quad \forall p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}, e > e(g)$$

Time-consistency property of ESD measure (c.)

- So, it happens that the *ESD measure is time-consistent*, since the following assertion is truth: $\tilde{z}_{ESD^g} = z_{ESD^g}$, for any node $g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$, any scenario tree, any profile set for any set period, and any other input data.
- Now, let us assume that the decisions in a given problem up to any of those nodes, say g, (i.e., the decisions in node set $\tilde{\mathcal{A}}^g$) have been made according to the solution obtained in the original model *ESD* 'solved' at period t=1.
- Then, the rationale behind a time-consistent risk averse measure is that the solution value to be obtained for the nodes in its successor set $\mathcal{S}^g$ in the scenario tree for the related submodel ESD^g 'solved' at stage $e(g)$ should have the same value as in the original model *ESD* 'solved' at period t=1.

Time-consistency property of ESD measure (c.)

- For practical reasons, the cardinality of the chosen stage set $e \in \mathcal{E}_f, f \in \mathcal{F}$ for risk reduction in the ESD measure should be small.
- Additionally, the profile $p \in \mathcal{P}_f^e$ could be different for the nodes g in set $\mathcal{G}^e, e \in \mathcal{E}_f, f \in \mathcal{F}$, provided that there are high differences in the objective function coefficients, al least, for the scenarios that belong to those different groups in the same stage.
- It is worth to point out that **the function values in function set $\mathcal{F}$ for risk reduction in model *ESD* refer to the scenarios in set Ω^g up to the end of the time horizon, for $g \in \mathcal{G}^e$.**

  On the other hand, **the risk reduction in model *TSD*, although referring to the same scenario group, it is only up to node g.**

Time-consistency property of ESD measure (c.)

- Since $\Omega_q \subset \Omega_g$ for $t(q) > t(g)$ and given the structure of ESD for risk reduction, it results that this measure, as any other ECRM with cross-node constraints,
- is very appropriate for considering the stochastic dynamic programming decomposition (SDP) methodology
  - introduced in Pereira & Pinto, WRR85, MP'91 as a suitable tool for solving large sized instances,
  - see also below and Cristobal, LFE, Monge, COR'09; LFE, Monge & Romero-Morales, TS'13, COR'15, COR'16 (submitted); and others.

Time-consistency property of ESD measure (c.)

- An interesting question: **What measure performs better for risk management either time-consistent or time-inconsistent?**
- Both address risk reduction at different time periods.

  A mixture of both is an ideal measure.

Time-consistency property of ESD measure (c.)

- An interesting question: **What measure performs better for risk management either time-consistent or time-inconsistent?**
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- **TYPES OF DECOMPOSITION METHODS FO0R STOCHASTIC OPTIMIZATION**

1. Benders Decomposition (BD) methodology (Benders, NM'62). Slyke & Wets SIAM'69 is the well-known first published algorithm in the subject. See also Aranburu et al. 2012; Lumbreras & Ranmos WN'13, among many others. The (nested) version for multistage problems, Birge MP'95.
2. Two-stage Lagrangean Decomposition (LD) heuristic methodology. See Caroe & Schultz 99; LFE et al., COR'13; Li & Ierapetritou, AIChE'12; Oliveira et al., SIPTO11; Oliveira et al., C&ChE13; Sagastizabal, MP'12, among many others. See also Gollmer-Neise-Schultz SIOPT'08; Gollmer-Gotzes-Schultz MP'11.
3. Multistage Clustering Lagrangean Decomposition (MCLD) heuristic methodology. See also LFE et.,al., COR'15; LFE et al., 2015a; 2015, LFE et., 2016; Mahlke 2011; Queiroz & Morton, ORL'13.

4. Regularization. See Asamov & Poowell arXiv'15, Mulvey & Ruszczynski, OR'95; Li & Ierapetritou, AIChE'12;, Rruszczynski MOR'95; Ruszczynski & Swietanowsk, SIOPT'97; Sen & Zhou, EJOR'14.
5. Progressive Hedging algorithm (PHA) for multistage primal decomposition was introduced in Rockafellar & Wets, MOR'91, Watson & Woodruff, CMS'11.
6. Multistage Stochastic Dynamic Programming (SDP). SDDP methodology, see Pereira & Pinto WRR'85, MP91; Ruszczynski, MP'93; LFE, Monge, Morales Tomero, TS'13; with CVaR risk averse measure see Aldasoreo et al., TOP'15; Cristobal, LFE, Monge COR'09; Guiges COAP'14; Kozmik & Morton, OptimizationOnlin'13; Shapiro et al., EJOR'13; and with SD LFE, Monge & Romero Morales COR'25, 2016.
7. Multistage cluster primal decomposition. LFE et al., COR'10, COR'12, EJOR'16; Mahlke 2011; Pages-Bernaus, Peres-Valdes & Tomasgard, EJOR'13; Sandikci, Kong &. Schaefer, MP'13; Zenarosa, 2014.

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- **BRIEF REF TO SOME DECOMPOSITION METHODS FOR MULTISTAGE STOCHASTIC SCENARIO TREE NODE CROSS CONSTRAINTS MIXED 0-1 RISK AVERSE PROBLEMS**

Exact multistage decomposition methods

- **Lagrangean lower bound and feasible solns providers**
  - **MCLD-RN** (Escudero, Garín & Unzueta, COR'15)
  - **MCLD-TSD** (Escudero, Garín & Unzueta, to be submitted March 2016)
  - **ENDO-MCLD-ESD** (Escudero, Garín, Monge & Unzueta, in preparation)
- **Exact Nested Benders**
- **Exact Branch-and-Fix Coordination:**
  - **BFC** risk neutral (Escudero, Garín, Merino & Pérez, COR'12)
  - **PC-BFC** (Aldasoro, Escudero, Merino & Pérez, COR'13)
  - **BFC-TSD** (Escudero, Garín, Merino & Pérez, EJOR'15)

- **ELP** (Beltrán-Royo, Escudero, Monge & Rodriguez-Revines, COR'14)
- **PC SDP** (Aldasoro, Escudero, Merino, Monge & Pérez, TOP'14)
- **SDP-SD** (Escudero, Monge & Romero-Morales, COR'15)
- **SDP-TSD/ESD** (Escudero, Monge & Romero-Morales, submitted 2016)
- **PC DBFC-RN** (Aldasoro, Escudero, Merino, Monge & Pérez, submitted 2015)
- **FRC-TSD**(Escudero, Garín, Pizarro & Unzueta, in preparation)

Contents

- **STOCHASTIC DYNAMIC PROGRAMMING (SDP) DECOMPOSITION MATHEURISTIC ALGORITHM**

SDP-ESD Introduction

- SDP-ESD, a SDP matheuristic algo, which combines consecutive stages in a set of stageblock, say $\mathcal{B}$, such that $\mathcal{E} = \cup_{b \in \mathcal{B}} \mathcal{E}^b$, $\mathcal{E}^b \cap \mathcal{E}^{b'} = \emptyset, b, b' \in \mathcal{B} : b \neq b'$, where $\mathcal{E}^b$ is the set of stages in block b. $B = |\mathcal{B}|$
- So, it decomposes the stochastic problem (35)-(36) into a collection of subproblems supported by scenarios subtrees as many as nodes in the original tree related to the first stage in the block.
- The subproblems in a block are linked to successor subproblems by the so-called *linking* decision variables.
- The immediate successor nodes to each leaf node in the scenario subtree that support the related subproblem in a given block are the root nodes of a set of scenario subtrees in the immediate consecutive stageblock to the given block.

SDP-ESD Introduction (c.)

- For solving the subproblems the concept of the so-called Expected Future Value (EFV) curves is used.
- Those curves estimate the impact of the linking decisions made at a given stageblock in the objective function value related to the future blocks.
- SDP-ESD is an iterative matheuristic where each iteration consists of a *forward* scheme followed by a *backward* scheme.

SDP-ESD Introduction (c.)

- The forward scheme is intended to improve the current solution, where the single subproblem in the first stageblock is solved and the linking variables are fed to subproblems in the second stageblock, which in turn are solved.

  This process is repeated until the last stageblock is reached, yielding a new solution.
- The backward scheme refines the current EFV curves by using this new solution.

  The EFV curves in the last stageblock are equal to zero since there are no future stageblock, thus the backward scheme starts in stageblock $|\mathcal{B}| - 1$.
- To refine those curves in stageblock $|\mathcal{B}| - 1$, strong duality theory is applied to the subproblems in stageblock $|\mathcal{B}|$ around the new solution.

  This process is repeated until first stageblock.

SDP-ESD Introduction (c.)

- The new proposal is designed to deal with model *ESD* and it is not a trivial task. Apart from the problem size that is allowed, another major challenge is to deal with the (probably, numerous) cross-scenario constraints that link the scenario groups g in a modeler-driven stage subset. The SD bounds are $\overline{d}^p$ and $\overline{\nu}^p$, for $g \in \mathcal{G}^e, p \in \mathcal{P}_f^e, e \in \mathcal{E}_f, f \in \mathcal{F}$, such that $e \in \mathcal{E}^B : e < E$.
- It is worth to point that, by construction, each subproblem to solve at any stageblock but the last one has not full information about the value of the objective function terms related to later stageblock, but an estimation.

  Notice that the later the stageblock as well as the deeper the iteration in the algorithm, the more precise the information is.

$\mathcal{G}^b \subseteq \mathcal{G}$, set of nodes in stageblock b, for $b \in \mathcal{B}$.

$\mathcal{R}^b \subseteq \mathcal{G}^b$, set of root nodes to the subtrees of stageblock b.

$\mathcal{C}^r \subseteq \mathcal{G}^b$, set of nodes in $\mathcal{G}^b$ that belong to the subtree rooted in node r, for $r \in \mathcal{R}^b, b \in \mathcal{B}$.

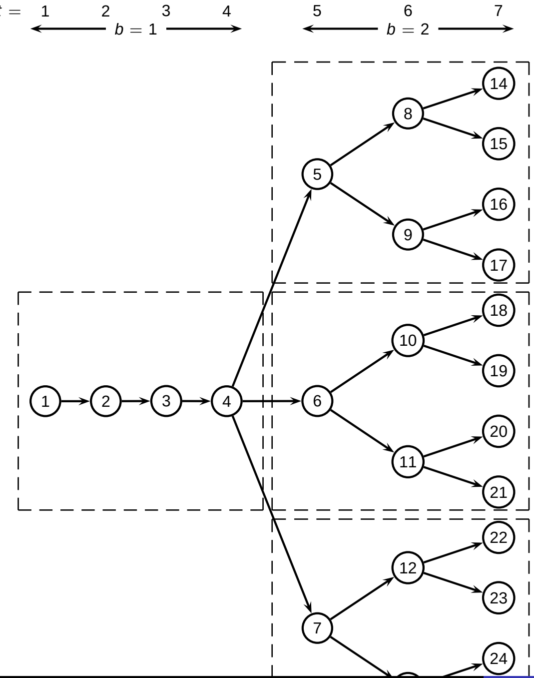
$\mathcal{L}^r \subseteq \mathcal{C}^r$, set of leaf nodes in $\mathcal{C}^r$, for $r \in \mathcal{R}^b$, $b \in \mathcal{B}$.

$\tilde{\tilde{\mathcal{A}}}^\ell \subseteq \mathcal{A}^\ell$, set consisting of leaf node $\ell \in \mathcal{L}^r$ and its ancestors, such that their variables have nonzero elements in constraints associated with the nodes in the immediate successor subproblems to node ℓ, defined by $\bigcup_{r' \in \mathcal{S}_1^e ll} C^{r'}$, for $\ell \in \mathcal{L}^r, r \in \mathcal{R}^b$, $b \in \mathcal{B} \backslash \{B\}$.

Definitions / Notation for the subproblems (c.)

Without loss of generality, let us assume that the node set $\{g \in \mathcal{G} : e(g) \in \mathcal{E}_f$ for performing risk reduction in the value of function f for $f \in \mathcal{F}$ is a subset of $\mathcal{C}_r$, $r \in \mathcal{R}^B$, i.e., only the nodes that belong to the stages in the last block can be a subject of risk reduction:

$$\mathcal{P}^f, \text{ set of profiles } \{p \in \mathcal{P}_f^e, e \in \mathcal{E}_f \cap \mathcal{E}^B : e < E\}.$$



$$\mathcal{G}^2 = \{5, \ldots, 25\}$$

$$\mathcal{R}^2 = \{5, 6, 7\}$$

$$\mathcal{C}^5 = \{5, 8, 9, 14, 15, 16, 17\}$$

$$\mathcal{L}^1 = \{4\}$$

Consider r for $r \in \mathcal{R}^B$.

- $\hat{x}^g, \hat{y}^{t,g}$: Given values of vectors $x^g, y^{t,g}$, for $t \in \mathcal{T}^{e(g)}, g \in \tilde{\mathcal{A}}^{\beta(r)}$.
- For $p \in \mathcal{P}_f^e, e \in \mathcal{E}_f \cap \mathcal{E}^B : e < E, f \in \mathcal{F}$:
- ESD variables:
  - $d^{\omega,p}$: Deficit (shortfall) of function f's value of scenario ω in set Ω^g for $e \equiv e(g)$.
  - $\nu^{\omega,p}$: Its value if $d^{\omega,p} > 0$ and otherwise, 0.
- ESD bounds:
  - $\overline{d}^p$ and $\overline{\nu}^p$.

EFV: $\lambda_{r'}^{'r}(\cdot)$ function for $r' \in \mathcal{S}_1^\ell$, $\ell \in \mathcal{L}^r$, $r \in \mathcal{R}^b$, $b \in \mathcal{B} \setminus \{B\}$

- It gives an approximation of the future value of function 1 in the set of scenarios $\Omega^{r'}$, related to the set of stageblocks $\{b' \in \mathcal{B} : b' > b\}$.

  It has the argument $(\cdot) = (x^g, y^{t,g}\, \forall t \in \mathcal{T}^{e(g)}, g \in \tilde{\tilde{\mathcal{A}}}^\ell)$.

Remember $\mathcal{P}^f$: set of profiles $\{p \in \mathcal{P}_f^e,\, e \in \mathcal{E}_f \cap \mathcal{E}^B : e < E\}$, for function $f \in \mathcal{F}$.

$$F_r'(\hat{x}^g, \hat{y}^{t,g} \forall t \in \mathcal{T}^{e(g)}, g \in \tilde{\tilde{\mathcal{A}}}^{\beta(r)}) =$$

$$\text{máx} \sum_{\ell \in \mathcal{L}^r} w_\ell [\sum_{g \in \tilde{\mathcal{A}}^\ell} (a_1^g x^g + \sum_{t \in \mathcal{T}^{e(g)}} b_1^{t,g} y^{t,g}) + (1-\rho^r) \sum_{r' \in \mathcal{S}_1^\ell} \lambda_{r'}^{\prime r}(\cdot)] -$$

$$\rho^r \sum_{f \in \mathcal{F}} \sum_{p \in \mathcal{P}^f} (M_D^p \varepsilon_D^p + M_{\underline{d}}^p \varepsilon_{\underline{d}}^p + M_{\underline{\nu}}^p \varepsilon_{\underline{\nu}}^p) \tag{37}$$

where $\rho^r = 1$ for $r \in \mathcal{R}^B$, otherwise, 0. subject to (41)-(46).

Remember $\mathcal{P}^f$: set of profiles $\{p \in \mathcal{P}_f^e,\ e \in \mathcal{E}_f \cap \mathcal{E}^B : e < E\}$, for function $f \in \mathcal{F}$.

$$\sum_{g' \in \mathcal{A}^g} \sum_{t' \in \mathcal{T}^{e(g')}: t' \leq t} (A_{t,g}^{g'} x^{g'} + B_{t,g}^{t',g'} y^{t',g'}) = h^{t,g} \quad \forall t \in \mathcal{T}^{e(g)},\ g \in \mathcal{C}^r \qquad (38)$$

$$x^g = \hat{x}^g,\ y^{t,g} = \hat{y}^{t,g} \qquad \forall t \in \mathcal{T}^{e(g)}, g \in \tilde{\mathcal{A}}^{\beta(r)} : t(r) > 1 \quad (39)$$

$$x^g \in \{0,1\}^{nx(g)}, y^{t,g} \in \mathbb{R}_+ \qquad \forall t \in \mathcal{T}^{e(g)}, g \in \mathcal{A}\ell, \ell \in \mathcal{L}_r \quad (40)$$

ESD^r subproblem supported by subtree whose nodes in $\mathcal{C}^r$ for root node $r \in \mathcal{R}^B$ of last stageblock B

Remember $\mathcal{P}^f$: set of profiles $\{p \in \mathcal{P}^e_f,\ e \in \mathcal{E}_f \cap \mathcal{E}^B : e < E\}$, for function $f \in \mathcal{F}$.

For $\forall p \in \mathcal{P}^f,\ f \in \mathcal{F}$:

$$\sum_{g' \in \tilde{\mathcal{A}}^\omega} (a_f^{g'} x^{g'} + \sum_{t' \in \mathcal{T}^{e(g')}})b_f^{t',g'} y^{t',g'}) + d^{\omega,p} \geq \phi^p \qquad \forall \omega \in \Omega^g : e = e(g) \tag{41}$$

$$d^{\omega,p} \leq D^p \nu^{\omega,p} + \varepsilon_D^p \qquad \forall \omega \in \Omega^g : e = e(g) \tag{42}$$

$$\sum_{\omega \in \Omega^g : e = e(g)} w^\omega d^{\omega,p} \leq \overline{d}^p + \varepsilon_{\overline{d}}^p \tag{43}$$

$$\sum_{\omega \in \Omega^g : e = e(g)} w^\omega \nu^{\omega,p} \leq \overline{\nu}^p + \varepsilon_{\overline{\nu}}^p \tag{44}$$

$$d^{\omega,p} \in \mathbb{R}_+,\ \nu^{\omega,p} \in \{0,1\} \qquad \forall \omega \in \Omega^g : e = e(g) \tag{45}$$

$$\varepsilon_D^p,\ \varepsilon_{\overline{d}}^p\, \varepsilon_{\overline{\nu}}^p \in \mathbb{R}_+ \tag{46}$$

- It is a piecewise linear convex function, as an approximation of the future value of function 1 in the set of scenarios $\Omega^{r'}$, to be obtained using strong duality theory at the *reference* levels.
- It gives an approximation of the future value of function f in the set of scenarios $\Omega^{r'}$, related to the set of stageblocks $\{b' \in \mathcal{B} \setminus \{B\} : b' > b\}$.
- Let $\mathcal{Z}^r_{r'}$ denote the set of reference levels, where the z-th one is included by vector

$$(\hat{X}^z_\ell;\ \pi^{q^z}_{r'}, \gamma^{(t,q)^z}_{r'}\ \forall t \in \mathcal{T}^{e(q)},\ q \in \tilde{\tilde{\mathcal{A}}}^\ell;\ \mu^z_{r'}), \tag{47}$$

where

$$\hat{X}^z_\ell \equiv (\hat{x}^{q^z}, \hat{y}^{(t,q)^z} \; \forall t \in \mathcal{T}^{e(q)}, \; q \in \tilde{\tilde{\mathcal{A}}}^\ell) \quad (48)$$

$\pi^{q^z}_{r'}, \gamma^{(t,q)^z}_{r'}$: the expected dual vectors of constraints (39):

$$x^g = \hat{x}^g, \; y^{t,g} = \hat{y}^{t,g} \quad \forall t \in \mathcal{T}^{e(g)}, g \in \tilde{\mathcal{A}}^{\beta(r)} : t(r) > 1$$

in subproblem (37)-(46) for $r' \in \mathcal{S}^\ell_1$.

$$\mu^z_{r'} = F_r(\hat{X}^z_\ell) - \sum_{q \in \tilde{\tilde{\mathcal{A}}}^\ell} \Big(\pi^{q^z}_{r'} \hat{x}^{q^z} + \sum_{t \in \mathcal{T}^{e(q)}} \gamma^{(t,q)^z}_{r'} \hat{y}^{(t,q)^z}\Big)$$

so that EFV curve $\lambda^r_{r'}$ can be expressed

$$\lambda^r_{r'} = \text{máx}_{z \in \mathcal{Z}^r_{r'}} \Big\{\mu^z_{r'} + \sum_{q \in \tilde{\tilde{\mathcal{A}}}^\ell} \Big(\pi^{q^z}_{r'} x^{q^z} + \sum_{t \in \mathcal{T}^{e(q)}} \gamma^{(t,q)^z}_{r'} y^{(t,q)^z}\Big) \quad (49)$$

Approximating ESD^r subproblem (37)-(46) supported by subtree whose nodes in $\mathcal{C}^r$ for root node $r \in \mathcal{R}^b$, $b \in \mathcal{B}$

Remember $\mathcal{P}^f$: set of profiles $\{p \in \mathcal{P}_f^e, e \in \mathcal{E}_f \cap \mathcal{E}^B : e < E\}$, for function $f \in \mathcal{F}$.

$$F_r(\hat{X}^r(48)) =$$
$$\text{máx} \sum_{\ell \in \mathcal{L}^r} w_\ell [\sum_{g \in \tilde{\mathcal{A}}^\ell} (a_1^g x^g + \sum_{t \in \mathcal{T}^{e(g)}} b_1^{t,g} y^{t,g}) + (1-\rho^r) \sum_{r' \in \mathcal{S}_1^\ell} \lambda_{r'}^r] -$$
$$\rho^r \sum_{f \in \mathcal{F}} \sum_{p \in \mathcal{P}^f} (M_D^p \varepsilon_D^p + M_{\underline{d}}^p \varepsilon_{\underline{d}}^p + M_{\underline{\nu}}^p \varepsilon_{\underline{\nu}}^p) \qquad (50)$$

subject to (51)-(54).

Approximating ESD^r subproblem (37)-(46) supported by subtree whose nodes in $\mathcal{C}^r$ for root node $r \in \mathcal{R}^b$, $b \in \mathcal{B}$ (c.)

Remember $\mathcal{P}^f$: set of profiles $\{p \in \mathcal{P}^e_f,\ e \in \mathcal{E}_f \cap \mathcal{E}^B : e < E\}$, for function $f \in \mathcal{F}$.

$$\text{s.t. Constraints (38) and (42)-(46)} \tag{51}$$

$$\lambda_f^{r'} \geq \mu_{r'}^z + \sum_{g \in \tilde{\tilde{\mathcal{A}}}^\ell} \Big(\pi^{q^z} x^{q^z} + \sum_{t \in \mathcal{T}^{e(q)}} \gamma_f^{(t,q)^z} y^{(t,q)^z}\Big)$$
$$\forall r' \in \mathcal{S}_1^\ell,\ \ell \in \mathcal{L}^r,\ z \in \mathcal{Z} : b < B \tag{52}$$

$$\sum_{q \in \tilde{\mathcal{A}}^\omega} (a_f^q x^q + \sum_{t \in \mathcal{T}^{e(q)}} b_f^{t,q} y^{t,q}) + d^{\omega,p} \geq \phi^p$$
$$\forall \omega \in \Omega^g : e = e(g),\ p \in \mathcal{P}^f,\ f \in \mathcal{F} : b = B \tag{53}$$

$$\lambda_\ell \in \mathbb{R} \quad \ell \in \mathcal{L}^r. \tag{54}$$

Rough idea of iterative matheuristic SDP-ESD

Remember $\mathcal{P}^f$: set of profiles $\{p \in \mathcal{P}_f^e,\, e \in \mathcal{E}_f \cap \mathcal{E}^B : e < E\}$, for function $f \in \mathcal{F}$.

- Each iteration of the SDP-ESD matheuristic consists of a forward scheme, followed by a backward scheme. See Escudero, Monge & Romero-Morales, COR'15 for the details.
- The forward scheme is aimed at building a solution for the original problem by solving models (51)-(54) $\forall r \in \mathcal{R}^b,\, b \in \mathcal{B}$.
- The backward scheme is aimed at refining the EFV curves around the solution $\hat{X}^r$ (48) built in that iteration.

  Subproblems from last stageblock B to 1st one1 are solved, passing the refinement of the EFV curves (52) onto the subproblems in the previous stageblock.
- EFV curve $\lambda^r_{r'}$, for $r' \in \mathcal{S}^\ell_1,\, \ell \in \mathcal{L}^r,\, r \in \mathcal{R}^b,\, b \in \mathcal{B} \setminus \{B\}$ is refined in a back-to-front scheme by adding a new reference level, and, then appending a new cons.

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