Short scientific report of STSM "Nonlinear network design" at Zuse Institute Berlin (ZIB), 11/08/14 - 15/08/14

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The purpose of this mission was to ammend the existing techniques for gas network optimization [4, 3, 5] to water networks, as specified in the submitted workplan. In particular, a preliminary formalization of the procedure for water networks had already been done by myself earlier this year. During the STSM, the intention was to develop a working implementation of these techniques together with Jesco Humpola and test it on benchmark instances for water networks.

Throughout the mission, we managed to get the data of the water network instances in a format similar to the one used in gas networks. We then obtained a working implementation of the procedure and tested it on two of the small test instances of the water network benchmark set (see, e.g., [2], the two instances were `shamir` and `hanoi`).

The existing techniques for topology optimization of gas networks [4, 3, 5] basically consist of three ingredients that can be incorporated into the branch-and-cut framework `SCIP` [1]: a heuristic method for finding feasible solutions, a way to cut off integer feasible nodes that are MINLP-infeasible by solving an abstractly convex nonlinear program, and the computation of cutting planes derived from dual information of this abstractly convex nonlinear program.

Regarding the optimal design of water distribution networks (WDND), all three ingredients can be extended in theory as well as implemented practically. However, on the two instances we tested the procedure, they turned out to be not as effective as expected. This is true especially for the third ingredient, the cutting planes, which we believe to be the most important. Instead, the heuristic and the cut-off mechanism worked without problems.

We were able to get an idea of what is the reason for the weakness of the cutting planes. In particular, a crucial difference seems to lie in the nature of the considered instances of gas networks, where the procedure turned out to be very effective [3], and the water network instances of the benchmark set. In the considered gas networks, typically the design decision to take is whether to build one or more parallel pipes with the same diameter as that of an existing pipe. For any number of such parallel pipes, one can compute the diameter of an equivalent single pipe. In theory, this situation is the one of the benchmark instances of water networks: on can choose one pipe of a finite set of pipes with different diameters. But the difference between water and gas is then the range between the minimum and the maximum diameter, which is higher for water networks. Also the increments in the diameter from one to the next index in the sequence of the available diameters is different. While in water networks, the diameter is typically a linear function in the index set of the available

diameters, in gas networks this relation is strictly concave.
This difference in the data of the instances results in the computed cut coefficients being too weak, and thus the cuts become ineffective. We tested this on two water network instances, but the intrinsic difference between the data of water and gas pursues throughout the whole benchmark set.

In order to remedy this flaw, more theoretical work is necessary. So far, the cutting planes are constructed in the following way. In a first step, a Benders-cut-type nonlinear valid inequality is established. This is the inequality that contains the important dual information of the abstractly convex leaf problem. However, the left-hand-side function of this inequality is nonlinear and unbounded from below. Therefore, in a second step, the function is modified such that it becomes bounded from below. In a third step, a linear underestimator for the modified function is computed. Our suspect is that the second and third step are responsible for the shortcomings described above. Therefore, a new way to handle the nonlinear valid inequality resulting from the first step has to be found.

For the future, we foresee to undertake more theoretical research in order to find such a new way. This will be done mostly by myself. Eventually, the implementation of a potential algorithm resulting from this will prefererably be done in conclusion with the Zuse-Institute Berlin. The intention is to publish a paper resulting from this collaboration and the STSM on an effective cutting plane procedure for water network design.
The STSM also offered the opportunity to discuss with other researchers of the Zuse-Instutute synergies between gas network optimization, which is extensively studied at the Zuse-Institute, and water networks, a subject well known to the research group at the University of Bologna. In particular, with Robert Schwarz, Ralf Lenz and Felipe Serrano I dicussed very vague ideas on continuous relaxations of nonlinear network design and tight convex relaxations of nonlinear flow constraints, possibly resulting in future collaborations.

References

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[3] A. Fügenschuh and J. Humpola. A new class of valid inequalities for nonlinear network design problems. Technical Report 13-06, Zuse-Institut Berlin, 2013.

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