Final Report of the STSM at Budapest University of Technology and Economics

#### **1** Purpose of the STSM

The main aim of this Short Term Scientific Mission was to collaborate with the Operations Research group at Budapest University of Technology and Economics, Institute of Mathematics. The idea was to join forces with two fields of expertise: (Mixed, non-)Linear complementarity problems ([M/N]LCP), Interior Point Methods, and Copositive Optimization. All problem classes are NP-hard, and they arise in a natural way in many fields of Energy Optimization (e.g. LCPs in modeling and optimization of Energy Markets and Production), and have several contacts in the underlying theory. In this short visit we identified possible research questions for common scientific work.

## 2 Description of the work carried out during the STSM

During this few days we discussed several different topics.

- The economics of power networks. This was the primal initial motivation of the STSM. In models of imperfect competition (Cournot models) among electricity producers, mixed-Linear Complementarity Problems (MLCPS) arise. These bilateral Nash-Cournot competition models may describe congestion pricing with or without arbitrage; in the latter case all prices which are not based on cost are leveled. An LCP arises if producers (naively) assume that their output does not affect transmission prices. However, if producers' decisions in fact do have an influence, more general nonlinear models are requested, along with efficient methods to solve them.
- 2. Ternary and Mixed-Binary Fractional Polynomial Optimization. Problems of this kind arise in chemical industry closely connected

to energy production (oil/gas processing, pooling/blending problems), but have many more applications, also in Network Optimization (max-cut-gain).

- 3. Lagrangian and Semi-Lagrangian bounds for quadratically and linearly constrained quadratic optimization. This problem class also includes mixed-integer models, and of course nonconvex instances arising in Energy Optimization.
- 4. Regarding solvability of LCPs (and NCPs), a central class of matrices is the so-called  $P_*$  class. Here we want to explore algorithmic possibilities to determine membership of a given matrix in this class, possibly building upon topic 3.

### **3** Description of the main results obtained

- 1. We studied the related literature and came to the following conclusion: (a) most of the articles focus on modeling, not on algorithms, since the instances treated seem to be small enough so as not to create too many problems; (b) the model assumptions are such that standard mathematical arguments suffice to ensure existence of the equilibria in question; (c) if more realistic models are applied to larger instances, both more elaborate mathematical arguments and refined algorithmic treatment involving Copositive Optimization may be needed, but as Economic Modeling is outside the scope of this STSM, we decided to involve specialists in this area for discussions before we proceed in our developments.
- 2. Triggered by our discussion and a small presentation, we decided to continue existing Copositivity Optimization approaches to achieve both, improved algorithms for a larger problem class, and basic research on the complexity of these problems (we conjecture APX-hardness, based on recent similar results) which still is an open problem.
- 3. This thread of joint research emerged from my seminar presentation at the host institution. We aim at comparing the presented methods with an alternative, new semi-Lagrangian approach using classical and very efficient trust-region methods for the subproblems. This comparison is very natural because both involve SDPs.
- 4. We established an alternative optimization formulation which is similar to bilevel QPs but where one level can be solved in closed form. For the other one, it is suggestive to employ methods addressed in

topic 3. We expect a procedure which systematically exploits easy instances of this NP-hard problem in an efficient way.

### 4 Future collaboration with the host institution

Given the close physical distance between the institutions of hosts and visitor, and the fruitfulness of the discussions (four topics on five days), intensive contacts and exchanges are foreseeable and easily to maintain.

### 5 Foreseen publications resulting from the STSM

I am quite optimistic that several papers will emerge in the near future (topics 2., 3., 4.), and their follow-ups, e.g. on topic 1.).

Budapest, 24th of May, 2014.

0 R Immanuel Bomze Visitor, University of Vienna

# Confirmation by the host institution of the successful execution of the STSM

I have read the final report written by Immanuel Bomze describing his STSM visit at Budapest University of Technology and Economics to collaborate with our research group. I am really satisfied with the visit, since lots of research questions are raised, which are very promising and may lead to important results.

Budapest, 24th of May, 2014.

GTON B

Boglárka G.-Tóth Host, Budapest University of Technology and Economics