SFB/Transregio TRR 154

Retreat Seeheim

Prof. Dr. Jens Lang

Problem Standardization in TRR154

14–16 March 2016

Overview

Euler equations: (TA1)

Conservation of mass, momentum and energy:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(p + \rho v^2) = -\frac{\lambda}{2D}\rho v\,|v| - g\rho h'$$

$$\frac{\partial}{\partial t}\left(\rho(\frac{1}{2}v^2 + e)\right) + \frac{\partial}{\partial x}\left(\rho v(\frac{1}{2}v^2 + e) + pv\right) = -\frac{k_w}{D}(T - T_w)$$

+ equation of state $p = R\rho T z(p, T)$

ρ **density**, p pressure, v velocity, T temperature, $m = \rho v$ **flux**,
$e = c_v T + gh$ inner energy, $E = \rho(\frac{1}{2}v^2 + e)$ **total energy**

see [Smoller1983, LeVeque1992, LeVeque2002, Brouwer2011]

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see [Smoller1983, LeVeque1992, LeVeque2002, Brouwer2011]

Real gas factor

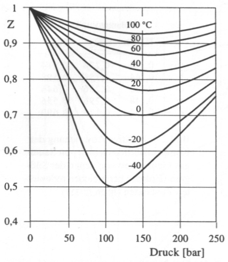
Equation of state: $p = R\rho T z(p, T)$

- perfect gas: $z = 1$
- real gas: $z = z(p, T)$ experimentally
- models for z
  f.e. AGA [Report No. 8]:

$$z(p, T) = 1 + 0.257\frac{p}{p_c} - 0.533\frac{pT_c}{p_c T}$$

- isothermal (T constant):

$$z(p) = 1 + \alpha p$$



[Rist1996]

[Osiadacz2010]: choice of equation of state has only minor impact on results

Friction coefficient λ

- depends on roughness k (in m) and Reynolds number $\text{Re} = \frac{\rho v D}{\eta}$ (η dynamic viscosity)
- laminar flow (Hagen-Poiseuille-flow)

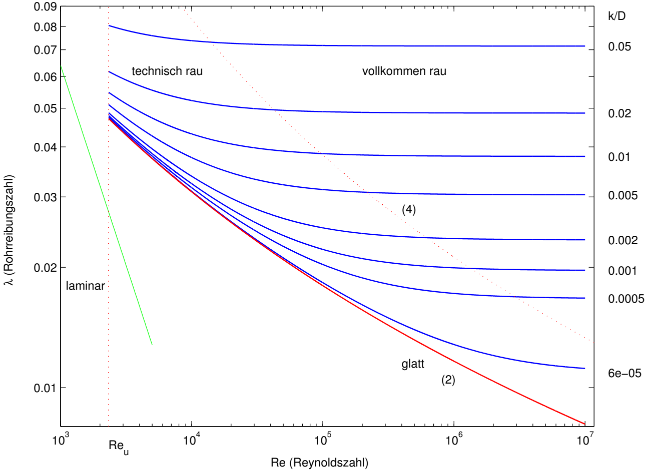
$$\lambda = \frac{64}{\text{Re}}$$

- turbulent flow: Colebrook's law (implicit)

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{2.5226}{\text{Re}\sqrt{\lambda}} + \frac{k}{3.7065\,D}\right)$$

- special cases
  $k = 0$ (hydraulically smooth pipe): Prandtl & Kármán,
  $\text{Re} \to \infty$ (totally rough pipe): Prandtl, Kármán & Nikuradse

Friction coefficient – Moody-Diagram



explicit approximate formulas

- **Weymouth** (1912):

$$\lambda = \frac{4}{\left(11.18 D^{1/6}\right)^2}$$

  Approximation for totally rough pipe with $k \approx 0.005\text{m}$
- **IGT-Equation** (Institute of Gas Technology)

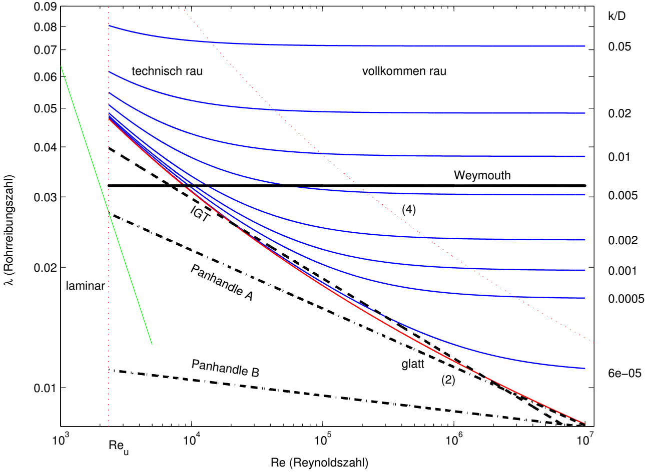
$$\lambda = \frac{4}{\left(4.619 \mathrm{Re}^{0.1}\right)^2}$$

  Approximation for hydraulically smooth pipe
- **Formula by Chen** (1979):

$$\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{\frac{k}{D}}{3.7065} - \frac{5.0425}{\mathrm{Re}}\log_{10}\left[\frac{\left(\frac{k}{D}\right)^{1.1098}}{2.8257} + \frac{5.8506}{\mathrm{Re}^{0.8981}}\right]\right)$$

  Approximation for overall turbulent region and all roughnesses

Friction coefficient – Moody-Diagram



Isothermal Models

Euler Equations **(TA1)**

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$
$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(p + \rho v^2) = -\frac{\lambda}{2D}\rho v\,|v| - g\rho h'$$
$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x}(vE + pv) = -\frac{k_w}{D}(T - T_w)$$

+ equation of state $p = R\rho T z(p, T)$

Eigenvalues: $v - c,\ v,\ v + c$

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Isothermal Models

Semilinear Equations **(ISO2)**

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$
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Eigenvalues: $-c$, $+c$

Isothermal Models

Semilinear Equations **(ISO2)**

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$
$$\frac{\partial}{\partial t}(\rho v) + \frac{\partial p}{\partial x} = -\frac{\lambda}{2D}\rho v\,|v| - g\rho h'$$

$+$ equation of state $p = R\rho T z(p)$

Eigenvalues: $-c$, $+c$

Isothermal Models

Semilinear Equations **(ISO3)** $\overset{\wedge}{=}$ (FD1) [Brouwer2011]

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0 \\ \frac{\partial p}{\partial x} = -\frac{\lambda}{2D}\rho v\,|v| - g\rho h' \end{aligned}$$

+ equation of state $p = R\rho T z(p)$

Eigenvalues: $\quad -\infty, +\infty$

Isothermal Models

Semilinear Equations **(ISO3)** $\triangleq$ (FD1) [Brouwer2011]

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

$$\frac{\partial p}{\partial x} = -\frac{\lambda}{2D}\rho v\,|v| - g\rho h'$$

+ equation of state $p = R\rho T z(p)$

Eigenvalues: $-\infty,\ +\infty$

Isothermal Models

Algebraic equations **(ISO4)**

$$\frac{\partial}{\partial x}(\rho v) = 0$$
$$\frac{\partial p}{\partial x} = -\frac{\lambda}{2D}\rho v\,|v|$$

resp. **(ISO-ALG)**: ρv constant in space and

$$p_{out} = \sqrt{p_{in}^2 - \frac{\lambda c^2 L}{2D}\rho v\,|\rho v|}$$

$\triangleq$ Weymouth-equation

Temperature-dependent Models

Euler Equations **(TA1)**

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

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+ equation of state $p = R\rho T z(p, T)$

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Temperature-dependent Models

(TA2)

$$\begin{aligned}
\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) &= 0 \\
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\frac{\partial}{\partial t}\left(\rho e\right) + \frac{\partial}{\partial x}\left(\rho v e + p v\right) &= -\frac{k_w}{D}\left(T - T_w\right)
\end{aligned}$$

+ equation of state $p = R\rho T z(p, T)$

Eigenvalues: $-c,\ v,\ +c$

Temperature-dependent Models

(TA2)

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$$

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Temperature-dependent Models

(TA3) $\triangleq$ (ET3) [Brouwer2011]

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Temperature-dependent Models

Stationary Model **(TA4)**

$$\begin{aligned}\frac{\partial}{\partial x}(\rho v) &= 0\\ \frac{\partial p}{\partial x} &= -\frac{\lambda}{2D}\rho v\,|v| - g\rho h'\\ \frac{\partial}{\partial x}\left(\rho v e + p v\right) &= -\frac{k_w}{D}\left(T - T_w\right)\end{aligned}$$

+ equation of state $p = R\rho T z(p, T)$

Temperature-dependent Models

Stationary Model **(TA4)**

$$
\begin{aligned}
\frac{\partial}{\partial x}(\rho v) &= 0 \\
\frac{\partial p}{\partial x} &= -\frac{\lambda}{2D}\rho v\,|v| - g\rho h' \\
\frac{\partial}{\partial x}(\rho v e + p v) &= -\frac{k_w}{D}(T - T_w)
\end{aligned}
$$

+ equation of state $p = R\rho T z(p, T)$

no gravitation, z constant, $\Rightarrow c^2 = \frac{p}{\rho}$ and $e = c_v T$

Temperature-dependent Models

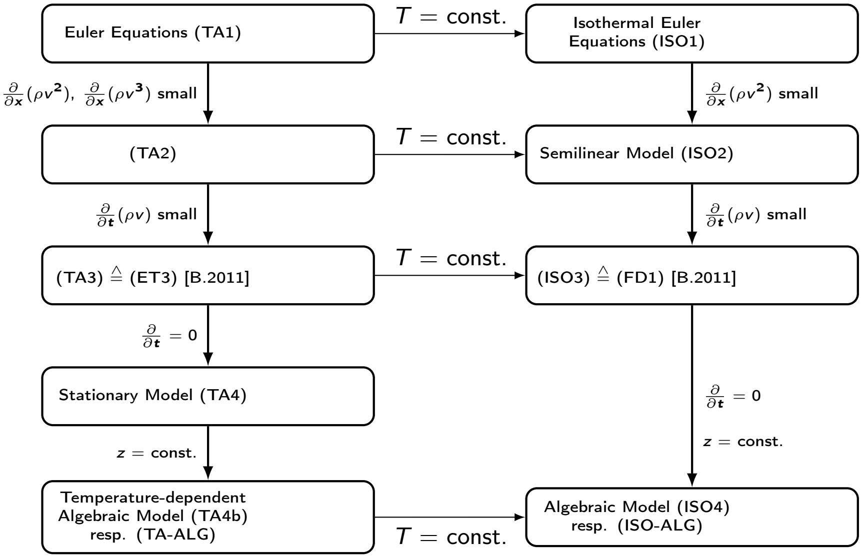
Temperature-dependent Algebraic Model **(TA4b)**

$$\frac{\partial}{\partial x}(\rho v) = 0$$
$$\frac{\partial p}{\partial x} = -\frac{\lambda c^2}{2Dp}\rho v\,|\rho v|$$
$$\frac{\partial T}{\partial x} = -\frac{k_w}{Dc_v\rho v}\left(T - T_w\right)\,.$$

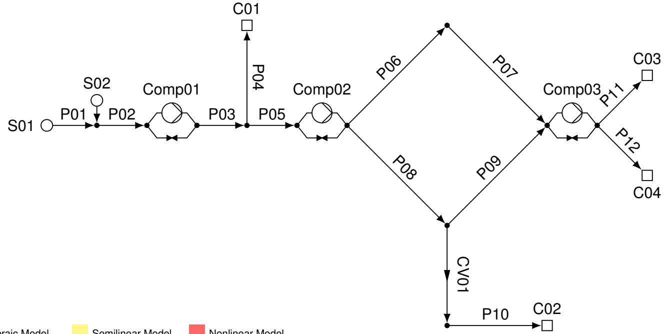
resp. **(TA-ALG)**: ρv constant in space and

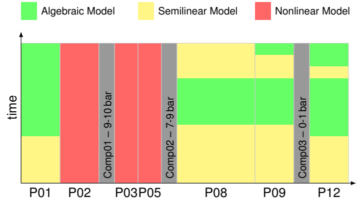
$$p_{out} = \sqrt{p_{in}^2 - \frac{\lambda c^2 L}{2D}\rho v\,|\rho v|}$$
$$T_{out} = \left(T_{in} - T_w\right)\cdot e^{-\frac{-k_w}{Dc_v\rho v}L} + T_w$$

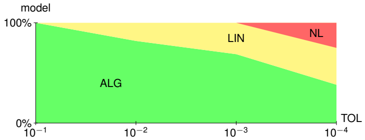
Model Hierarchy



Model Adaptivity in Gas Networks: E.ON





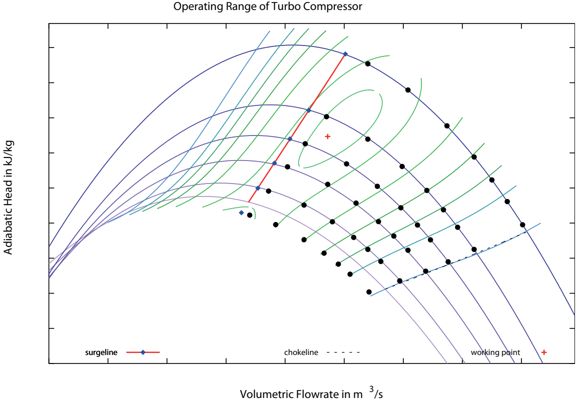


Network Elements

- pipe
- valve: on or off
- nonreturn valve
- resistance (describes pressure loss)
- stabilizer (regulates flow rate w.r.t. pressure)
- pre-heater and cooler (as part of a compressor station)
- compressor and compressor groups

Compressor

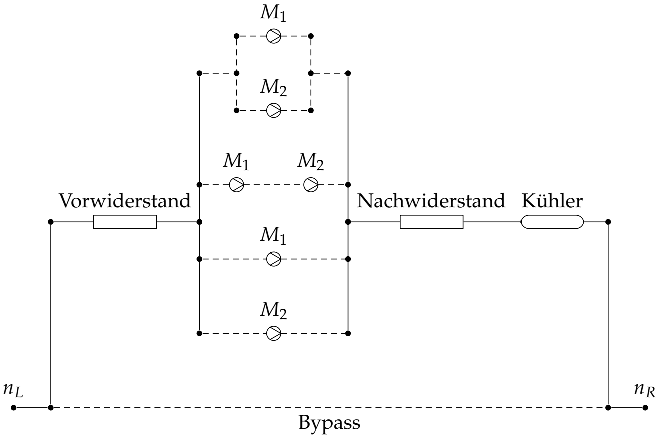
Turbo-compressors are modeled by characteristic diagrams.



blue lines = constant speed, green lines = constant efficiency

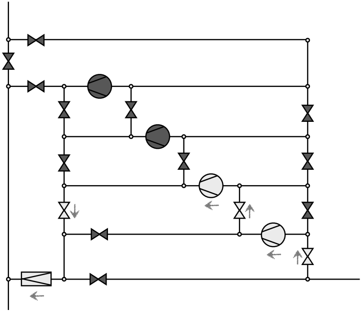
Compressor Group

Unit of compressors, pre-heaters, coolers, resistances, ...



Compressor Group

Different options for compression



Network Modeling

- Network = directed graph with edges and nodes
- Coupled differential-algebraic system for network dynamics

  – hyperbolic PDEs

  conservation/balance of mass, momentum and energy in pipes

  – algebraic and ODEs

  compressors, valves, heaters, coolers, ...

  – purely algebraic

  coupling and boundary conditions
- Continuous and discrete control variables

  – discrete: compressor on/off, valve open/closed

  – continuous: electric power for compressors and pumps