Modelling Uncertainty in Transmission System Investment Planning with Transmission Switching

Jakub Marecek
with Martin Mevissen, Mathieu Sinn, Jonas Christoffer Villumsen
IBM Research - Ireland

PGMO-COST Workshop on Validation and Software, October 27th 2014

SmarterCities
Technology Centre

Overview

- **Part 1:** The motivation
- **Part 2:** Characterising uncertainty
- **Part 3:** Network expansion problem with transmission switching
- **Part 4:** Demo

The many approaches to optimisation under uncertainty I

- To paraphrase von Neumann:
  uncertainty characterisation is like a non-elephant
- There are very many approaches to optimisation under uncertainty
- Even if you do the distributional forecasting well,
  you get one solution for stochastic programming with 3 scenarios, one for stochastic programming with 4 scenarios, one solution for the usual robust optimisation, one for Bertsimas-Sim with the budget $\Gamma = 3$, one with the budget $\Gamma = 4$, ...
- The solutions can be evaluated with respect to a number of key performance indicators (KPIs), but none very likely dominates all others.
- You need to pick exactly one.

The quote on control and elephants is attributed to John von Neumann.

The many approaches to optimisation under uncertainty II

There are a number of varieties in optimisation under uncertainty:

- Aggregation of data, e.g. consider ranges corresponding to supports of probability distributions
- Aggregation of solutions, e.g. average over the solutions obtained for each scenario, deterministically
- Hybrids, e.g. take a small range around the values in each scenario, average over the solutions thus obtained

which proceed in two steps:

- You solve one or more instances of (integer) convex deterministic optimisation, a.k.a. robust counterparts, scenario expansions
- You map the solutions of the deterministic instance(s) back to solutions of the problem in optimisation under uncertainty.

The many approaches to optimisation under uncertainty III

Ref.	Authors	Captures
	Aggregation-of-Data Approaches (aka. Robust)	
[14]	Soyster	Interval uncertainty for inequalities
[1]	Ben-Tal-Nemirovski	Ellipsoidal uncertainty
[10]	ElGhaoui et al.	Worst-case value-at-risk (VaR)
[3]	Bertsimas-Sim	L_0 budget of uncertainty
[9]	Fischetti-Monaci	Budget of uncertainty wrt. the objective
	Sampling-of-Data Approaches (aka. Stochastic)	
[7]	Dembo	Sample and optimise
[6]	Dantzig	Two-stage stochastic opt.
[5]	Campi-Garatti	Chance-constrained opt.

The many approaches to optimisation under uncertainty IV

Ref.	Authors	Captures
	Hybrid approaches	
[12]	Mulvey-Vanderbei-Zenios	Penalty for cons. violation
[4]	Möhring et al	Recoverable Robustness
	Calafiore-El Ghaoui	Distributional robustness
[13]	Natarajan-Pachamanova-Sim	Discrete Conditional VaR
[8]	Fischetti-Monaci	Light Robustness
[2]	Ben-Tal et al.	Soft robustness
[2]	Ben-Tal et al.	Comprehensive robustness
[11]	Mareček	Mixed criticality

Since 2013

IBM Software:

- IBM ILOG CPLEX is a library of optimisation routines
- IBM ILOG Decision Optimisation Center is a “platform for building and deploying analytical decision support applications”. It connects data in a SQL database to an algebraic model, such that the solving is performed remotely, in a distributed fashion, and such that custom user interfaces are easy to develop.

N.B. Decision Optimisation Center will be presented by Alex Fleischer.

A 2013 joint program of IBM Research and Decision Optimization:

- A user-friendly toolkit as plug-in to Decision Optimization Center
- Built-in automated reformulation

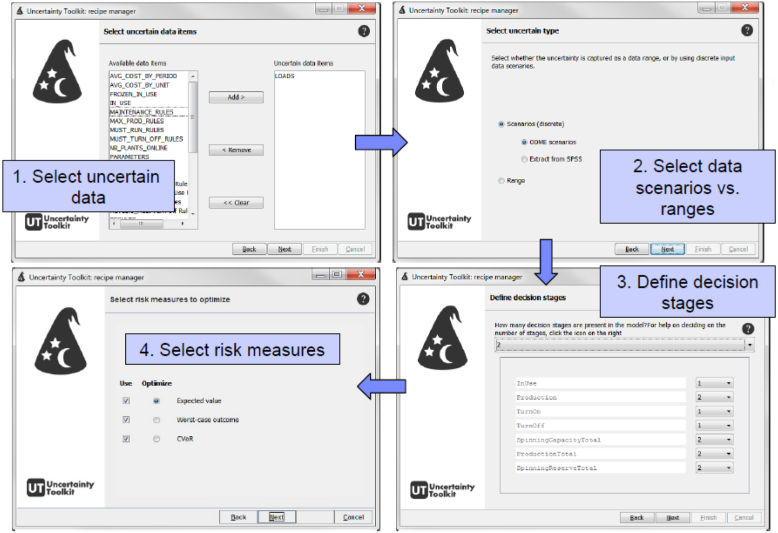
Information about the joint programme are quoted from [Kawas et al., IFORS 2014].

The Key Idea

- The 'recipe' captures the uncertainty characterization for a given deterministic algebraic optimization model
- Automated reformulation of the deterministic algebraic optimization model based on the 'recipe' and a number of scenarios.
- Using a single deterministic algebraic optimization model and one set of scenarios, multiple 'recipes' give you multiple solutions hedging against uncertainty in various ways.

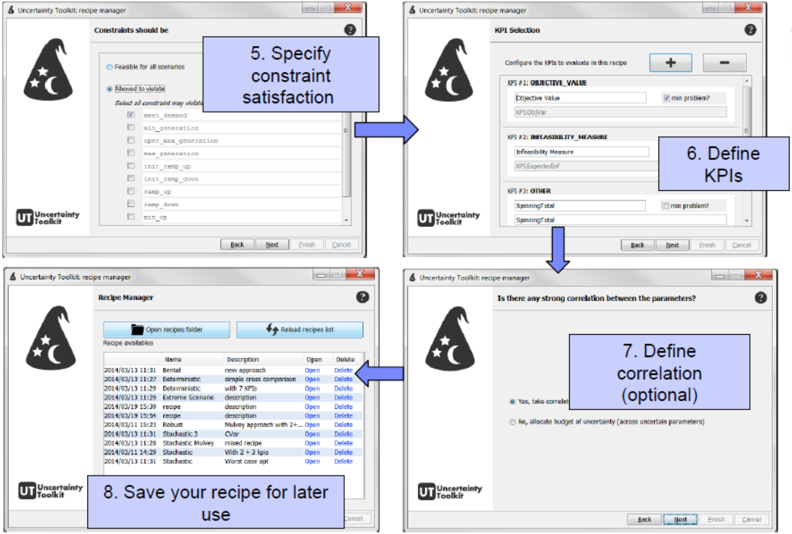
- An advanced user can devise the “recipe” using a wizard
- Any user apply use “the recipe” using another wizard
- Any user can compare multiple solutions hedging against uncertainty

The Wizard I



From the presentation of [Kawas et al., IFORS 2014]

The Wizard II



From the presentation of [Kawas et al., IFORS 2014]

The Investment Planning

- OECD: Global infrastructure spending is estimated at USD 2.2T p.a.
- EDF invested circa EUR 12B, while RTE invested EUR 1.45B in 2013

We consider:

- Investment into switching equipment and line capacity in an electricity distribution system employing dynamic reconfiguration of the network topology
- Minimisation of capital and operational cost with respect to uncertain load and generation scenarios
- Maximisation of reliability with respect to line failures
- A two-stage problem with discrete decisions at both stages.

Investment figures from 2013 annual reports of EDF and RTE

The Investment Planning: The Objective

- Amortised investment costs f_L for line capacity
- Amortised investment costs f_S for switching equipment
- Probability π_k of scenario $k \in K$
- Operational decisions Q_k feasible in scenario $k \in K$
- Operational costs c_k in scenario $k \in K$
- Reliability r_k in scenario $k \in K$ (e.g. SAIFI)

$$\min f_S x_S + f_L x_L + \sum_{k \in \mathcal{K}} \pi_k \left(c_k^\top q_k + r_k(q_k) \right) \tag{1}$$

$$\text{s.t. } x_L - x_S + y_k \leqslant 1 \quad \forall k \in \mathcal{K} \tag{2}$$

$$x_L + y_k \geqslant 1 \quad \forall k \in \mathcal{K} \tag{3}$$

$$(y_k, q_k) \in \mathbb{Q}_k \quad \forall k \in \mathcal{K} \tag{4}$$

$$x_S, x_L \in \{0, 1\}^{|A|} \tag{5}$$

The Investment Planning: The Constraints $\mathbb{Q}_k$

- A steady-state model within each time period, i.e. with constant current injection and loads
- A rudimentary piece-wise linearisation of the ACOPF, i.e. linear outer approximations and disjunctive constraints

Formulation 1 (based on Ferreira et al., 2013):

- Rectangular current-voltage formulation with switchable lines
- Power injection $\widehat{p} + \mathbf{j}\widehat{q}$
- Current injection $I = \widehat{p}\mu + \widehat{q}\nu + \mathbf{j}(\widehat{q}\mu - \widehat{p}\nu)$ where $\mu = \frac{v}{v^2+u^2}$ and $\nu = \frac{u}{v^2+u^2}$.

Formulation 2 (based on Trodden et al., 2013):

- Polar power-voltage formulation with switchable lines
- Linearised sine function and piece-wise linearised cosine

The Investment Planning: Formulation 1

- Piece-wise linearisation at evaluation points $(\widehat{v}, \widehat{u})$ with values μ, ν:

$$v_i = \sum_{j,k} \widehat{v}_i^j \lambda_i^{jk} \qquad u_i = \sum_{j,k} \widehat{u}_i^k \lambda_i^{jk} \qquad e^\top \lambda = 1$$

$$\mu_i = \sum_{j,k} \widehat{\mu}_i^{jk} \lambda_i^{jk} \qquad \nu_i = \sum_{j,k} \widehat{\nu}_i^{jk} \lambda_i^{jk}$$

- The SOS2 constraints:

$$\lambda^{jk} \leqslant \psi^{j-1} + \psi^j, \forall j \in \mathcal{J} \setminus \{0\}$$

$$\lambda^{jk} \leqslant \chi^{k-1} + \chi^k, \forall k \in \mathcal{K} \setminus \{0\}$$

$$\lambda^{j,0} \leqslant \psi^0 \qquad \lambda^{0,k} \leqslant \chi^0$$

$$e^\top \psi = e^\top \chi = 1 \qquad \psi, \chi \in \{0,1\}$$

where $\mathcal{J}$ is the index set of points $\widehat{v}$ and $\mathcal{K}$ is the index set of points $\widehat{u}$. $\psi^j = 1$ iff v is in $\left[\widehat{v}^j, \widehat{v}^{j+1}\right]$, and $\chi^k = 1$ iff u is in $\left[\widehat{u}^k, \widehat{u}^{k+1}\right]$

The Investment Planning: Formulation 2

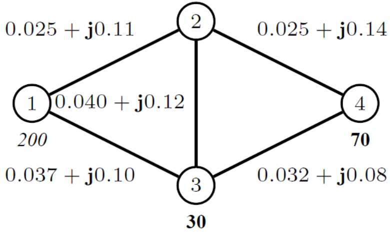
Power balance equations in the polar formulation:

- $P_i = +\sum_{k=1}^{N} |V_i||V_k|(G_{ik}\cos\theta_{ik} + B_{ik}\sin\theta_{ik})$
- $Q_i = \sum_{k=1}^{N} |V_i||V_k|(G_{ik}\sin\theta_{ik} - B_{ik}\cos\theta_{ik})$

Linearised sine function and piece-wise linearised cosine:

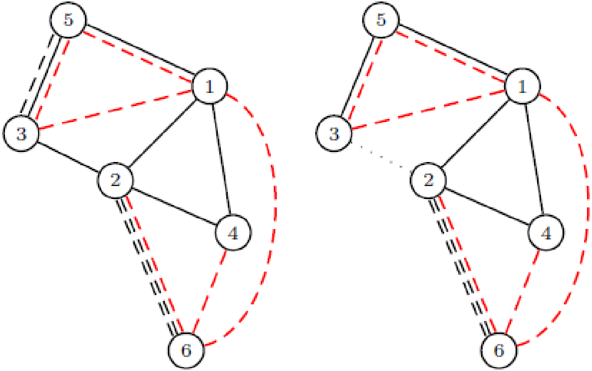
- $\sin(\theta_{ij})$ linearised by θ_{ij}
- $\cos(\theta_{ij})$ piece-wise linearised (instead of 1)
- V_i^2 linearised to $2v_i - 1$
- $v_i v_j \cos(\theta_{ij})$ approximated by $v_i v_j + \cos(\theta_{ij}) - 2$
- $v_i v_j \sin(\theta_{ij})$ approximated by θ_{ij}

Illustrations: 4-Bus Network, AC v. PWL

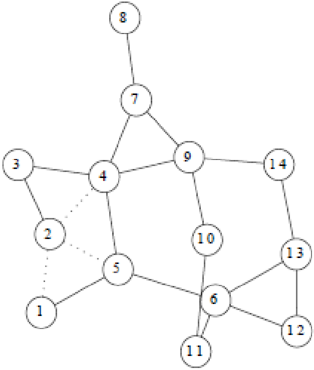


Configuration (lines)	ACOPF cost	ACOPF rank	PWLOPF cost	PWLOPF obj.	PWLOPF rank
all	102.12	1	102.60	-8.43	1
all except 2-3	102.18	2	102.63	-8.38	2
all except 3-4	102.48	3	103.03	-7.70	3
all except 2-4	103.56	4	104.33	-6.77	4
all except 1-3	103.83	5	104.77	-5.73	6
all except 1-2	104.38	6	105.36	-5.75	5
1-3, 2-3, 2-4	106.95	7	108.20	-1.02	7
1-2, 2-3, 3-4	108.38	8	109.75	1.52	8

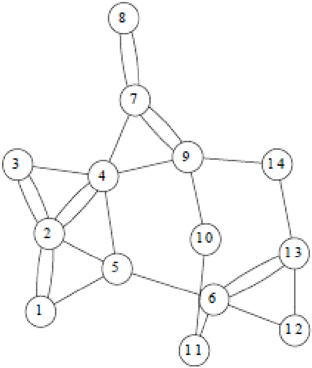
Illustrations: Graver's Network, PWL v. DC



Illustrations: IEEE 14-Bus, Optimum Switching



Illustrations: IEEE 14-Bus, PWL Expansion vs. 0 for DC



The Demo

The Conclusions

- It is important to know what uncertainty characterisation works best
- ... but it is hard to learn that without the appropriate tools.

Further References I

A. Ben-Tal and A. Nemirovski.
Robust solutions of uncertain linear programs.
Operations research letters, 25(1):1–13, 1999.

Aharon Ben-Tal, Dimitris Bertsimas, and David B. Brown.
A soft robust model for optimization under ambiguity.
Oper. Res., 58(4-Part-2):1220–1234, July 2010.

Dimitris Bertsimas and Melvyn Sim.
The price of robustness.
Operations Research, 52(1):35–53, 2004.

Christina Büsing, ArieM.C.A. Koster, and Manuel Kutschka.
Recoverable robust knapsacks: the discrete scenario case.
Optimization Letters, 5:379–392, 2011.

M.C. Campi and S. Garatti.
A sampling-and-discarding approach to chance-constrained optimization: Feasibility and optimality.
Journal of Optimization Theory and Applications, 148:257–280, 2011.

George B. Dantzig.
Linear programming under uncertainty.
Management Science, 1(3-4):197–206, 1955.

Ron Dembo.
Scenario optimization.
Annals of Operations Research, 30:63–80, 1991.
10.1007/BF02204809.

Further References II

Matteo Fischetti and Michele Monaci.
Light robustness.
In Ravindra K. Ahuja, Rolf H. Möhring, and Christos D. Zaroliagis, editors, *Robust and Online Large-Scale Optimization: Models and Techniques for Transportation Systems*, volume 5868 of *Lecture Notes in Computer Science*, pages 61–84. Springer, 2009.

Matteo Fischetti and Michele Monaci.
Cutting plane versus compact formulations for uncertain (integer) linear programs.
Mathematical Programming Computation, 4:239–273, 2012.

Laurent El Ghaoui, Maksim Oks, and Francois Oustry.
Worst-case value-at-risk and robust portfolio optimization: A conic programming approach.
Oper. Res., 51(4):543–556, July 2003.

Jakub Marecek.
Optimization of Mixed-Criticality Systems.
Patent application serial number US 14/286004., 2014.

John M. Mulvey, Robert J. Vanderbei, and Stavros A. Zenios.
Robust Optimization of Large-Scale Systems.
Operations Research, 43(2):264–281, 1995.

Karthik Natarajan, Dessislava Pachamanova, and Melvyn Sim.
Constructing risk measures from uncertainty sets.
Oper. Res., 57(5):1129–1141, September 2009.

Further References III

A. L. Soyster.
Convex programming with set-inclusive constraints and applications to inexact linear programming.
Operations Research, 21(5):pp. 1154–1157, 1973.

Backup: IEEE 14-Bus, Accuracy

	1x4x1x1		2x4x1x1		1x8x1x1		1x4x2x1		1x4x1x2		1x8x1x2		1x16x1x2		ACOPF(MP)		DCOPF(MP)	
bus	p	q	p	q	p	q	p	q	p	q	p	q	p	q	p	q	p	q
1	15.382	0	15.358	0	20.263	10.007	15.346	0	17.785	6.872	21.855	0	24.551	0	25.34	0.12	19	0
2	140.043	49.815	140.035	48.89	140.194	49.298	140.109	49.846	140.11	23.625	140.088	37.625	140.153	35.435	140	20.68	140	0
3	5.927	26.497	5.984	27.378	2.533	20.324	5.969	25.83	4.323	34.545	1.658	29.634	0.224	35.999	0	32.1	0	0
4	0.77	-0.58	0.826	-0.239	0.223	0.191	0.761	-0.554	0.817	-0.625	0.262	0.142	-0.015	0.196	0	0	0	0
5	0.085	-0.033	0.084	0	0.03	-0.041	0.083	-0.031	0.095	-0.037	0.041	-0.051	0.003	-0.025	0	0	0	0
6	99.993	1.45	99.714	1.014	100.532	3.705	99.972	2.138	99.757	10.103	100.269	10.819	100.444	12.652	100	24	100	0
7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	1.635	18.793	1.635	18.771	1.375	14.093	1.647	18.799	0.857	20.885	0.722	18.354	0.322	12.712	0	18.56	0	0
9	0.796	0.144	0.882	0.182	0.282	-0.012	0.79	0.17	0.844	0.093	0.32	-0.142	-0.041	-0.081	0	0	0	0
10	0.203	0.062	0.225	0.074	0.078	0.013	0.202	0.069	0.219	0.048	0.089	-0.023	0.011	-0.041	0	0	0	0
11	0.018	0.002	0.026	0.006	0.006	-0.003	0.018	0.003	0.024	0	0.009	-0.008	0	-0.013	0	0	0	0
12	0.003	-0.008	0.019	0.003	-0.008	-0.01	0.004	-0.008	-0.011	-0.006	-0.02	-0.006	-0.021	-0.006	0	0	0	0
13	-0.032	-0.017	-0.006	-0.004	-0.032	-0.014	-0.03	-0.016	-0.025	-0.017	-0.026	-0.023	-0.03	-0.028	0	0	0	0
14	0.305	0.049	0.326	0.054	0.124	0.026	0.308	0.059	0.317	0.019	0.116	-0.033	0.01	-0.051	0	0	0	0
bus	$\|V\|$	ϑ	$\|V\|$	ϑ	$\|V\|$	ϑ	$\|V\|$	ϑ	$\|V\|$	ϑ	$\|V\|$	ϑ	$\|V\|$	ϑ	$\|V\|$	ϑ	$\|V\|$	ϑ
1	1.0551	0	1.0551	0	1.0535	0	1.0501	0	1.0526	0	1.053	0	1.0521	0	1.06	0.000*	1	0.000*
2	1.0595	-0.05	1.0595	-0.04	1.0532	-0.09	1.0545	-0.04	1.0517	0	1.0555	-0.17	1.0543	-0.24	1.06	-0.185	1	-0.037
3	1.0367	-6.51	1.0377	-6.53	1.0103	-6.52	1.031	-6.49	1.041	-6.83	1.0256	-6.95	1.0254	-7.23	1.032	-6.757	1	-6.937
4	1.0337	-3.18	1.034	-3.19	1.0186	-3.24	1.0287	-3.18	1.0357	-3.37	1.0299	-3.52	1.0252	-3.61	1.039	-3.508	1	-3.668
5	1.0372	-1.84	1.0374	-1.84	1.0262	-1.91	1.0324	-1.85	1.039	-2.02	1.0362	-2.16	1.0329	-2.25	1.047	-2.226	1	-2.289
6	1.0053	1.96	1.0048	1.94	0.9945	1.84	1.0014	1.94	1.0217	1.65	1.0189	1.47	1.0166	1.32	1.053	0.857	1	1.206
7	1.016	-4.05	1.0162	-4.06	0.9938	-4.09	1.0113	-4.05	1.0247	-4.29	1.0139	-4.44	1.0018	-4.5	1.029	-4.478	1	-4.574
8	1.0479	-4.14	1.0481	-4.15	1.0183	-4.15	1.0434	-4.14	1.06	-4.5	1.0451	-4.64	1.0237	-4.66	1.06	-4.478	1	-4.574
9	0.9868	-4.43	0.9869	-4.44	0.9655	-4.47	0.9821	-4.43	0.9968	-4.63	0.9861	-4.77	0.9758	-4.83	1.005	-5.005	1	-5.051
10	0.9795	-3.58	0.9796	-3.59	0.9606	-3.64	0.9749	-3.59	0.9912	-3.8	0.9822	-3.95	0.9735	-4.03	1.004	-4.247	1	-4.368
11	0.9815	-1.04	0.9814	-1.05	0.9697	-1.14	0.9772	-1.05	0.9966	-1.3	0.9934	-1.47	0.9891	-1.6	1.022	-1.818	1	-1.826
12	0.9848	0.55	0.9844	0.54	0.9758	0.43	0.9807	0.54	1.0009	0.25	1.0001	0.08	0.9987	-0.07	1.036	-0.398	1	-0.337
13	0.9752	0.07	0.9748	0.06	0.9665	-0.03	0.971	0.06	0.9923	-0.2	0.9919	-0.37	0.9899	-0.51	1.026	-0.808	1	-0.844
14	0.9645	-3.42	0.9645	-3.43	0.9466	-3.5	0.96	-3.42	0.978	-3.65	0.9696	-3.81	0.9618	-3.91	0.994	-4.234	1	-4.511
loss	6.12836	22.6732	6.10797	22.6303	6.6008	24.0778	6.1786	22.8042	6.1109	22.0047	6.38438	22.7877	6.61116	23.2483	6.341	21.950	0.000	0.000
obj	248.908		249.117		246.163		249.065		245.093		243.307		241.676		240.68		228	
solve time	1.13	-	2.97		2.59		1.45		2.17		6.41		61.87		0.12		0.06	