A scheme for solution validation in mathematical optimization under uncertainty

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PGMO-COST Workshop on Validation and Software

EU COST-Action TD1207

27 October 2014, ENSTA Paris Tech, Palaiseau, France

Universidad Rey Juan Carlos

UPV EHU

Contents

Illustrative case for energy planning: EGCEP

- Assume a model for addressing challenges for a long term (e.g., 30 years) power generation capacity expansion planning.
- Scope: Helping to decision making on:
  1. Type and mix of power generation sources (ranging from coal, nuclear and combined cycle gas turbine to more renewable sources: hydroelectric, wind, solar, photovoltaic and biomass)
  2. New power generation plant / farm location and capacity

Illustrative case for energy planning: EGCEP. Characteristics

- Uncertainty, Multicriteria and Nonlinearity in EGCEP
- A gigantic but well structured multicriteria multistage SMINO problem with risk management.
  - Dynamic setting
  - Site location and capacity decisions
  - Replicated networks (hydro valleys). Eg., 20+ valleys, some with 50 elements, see S. Charousset, COST WMINLP, Paris, 2013.
  - General networks: (current and candidate power generation plants, energy demand node)
- Algorithmic framework for MINO under uncertainty in dynamic setting, see LFE et al., WMINLP, Pittsburgh, 2014.
- SMINO in Electricity Generation, see S. Charousset, COST WMINLP, Paris, 2013.
- Assumption: Transmission network is not a constraint. Otherwise, see EGCTEP in LFE et al., COST Workshop, Budapest, 2014.

Uncertainty in EGCEP. Main parameters

- Market electricity demand and prices at the network nodes of the energy system.
- Raw material cost and availability: Fuel, Water exogenous inflow, Wind, Solar intensity.
- Operating hours per period of power generation technologies.
- CO_2 emission permits and Green Certificates prices and allowed bounds.
- Power generation fixed and variable cost of different technologies

- Electricity loss of new transmission technologies.
- Characteristics (i.e., maximum energy flow and reactance) of cable types on new energy transmission lines.
- Fixed and variables costs of energy transmission technologies.

Uncertainty in EGCEP. Representation

- A stage in time horizon: Consecutive years whose constraint systems must be satisfied in an individual basis).
- Multistage non-symmetric scenario tree
- It is required a 'must' combination of:
  - Sample scenario schemes
  - SMINO $\rightarrow$ Sequential SMILO
  - inexact scenario group Decomposition algos
  - High Performance Computing

Multicriteria in EGCEP

- Maximizing NPV of expected investment and consumer stakeholders goals over the scenarios along the time horizon subject to risk reduction of the negative impact of non-wanted scenarios on multiple types of utility objectives and stakeholders:
    - Maximizing power share of cleaner, safer and efficient -cheaper- energy accessible to all consumption nodes.
    - EC directives on environmental issues and others.
    - EU governments, etc.

EGCEP. Aim

- A mathematical optimization model may help to determine the evolution of the power generation mix and location along a time horizon. So, it determines for every power generation technology:
  - **site location of each new power plant**
  - **year to start the construction**

  depending on the scenario group (i.e., node in the scenario tree) along the time horizon.
- It has to be considered that at the year when the new power plant is ready for being in operation, **the realization of uncertain prices and electricity demand and other key uncertain elements can be drastically changed along the scenario tree from the scenario group where the construction have started.**

Expected Value (deterministic) mixed 0-1 optimization model

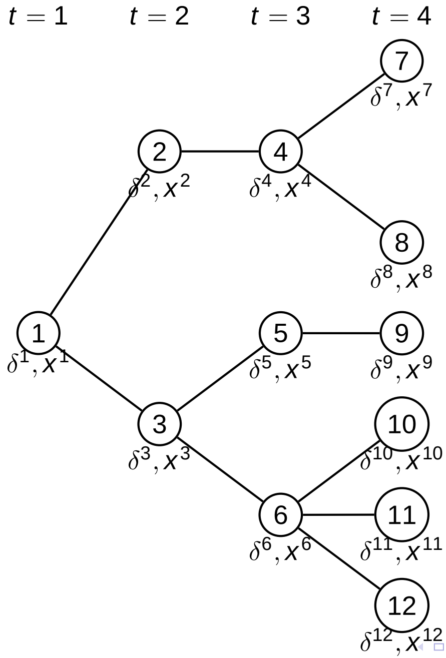
$$
\begin{aligned}
z_{EV} &= \max \sum_{t\in\mathcal{T}} F^t(\delta^t, x^t) \\
\text{s.t.} \sum_{t'\in\mathcal{A}^t} f^t(\delta^{t'}, x^{t'}) &= h^t \qquad \forall t \in \mathcal{T} \\
\delta^t \in \{0,1\}^{n(t)},\ x^t &\in \mathbb{R}^{n(t)} \quad \forall t \in \mathcal{T}.
\end{aligned} \tag{1}
$$

Multistage scenario tree

A **stage** of a given horizon is a set of consecutive time periods where the realization of the uncertain parameters takes place.

A **scenario** is a realization of the uncertain parameters along the stages of a given horizon.

A **scenario group** for a given stage is the set of scenarios with the same realization of the uncertain parameters up to the stage.



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t = 2
t = 3
t = 4
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δ2, x2
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δ3, x3
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δ4, x4
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δ5, x5
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δ6, x6
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δ11, x11
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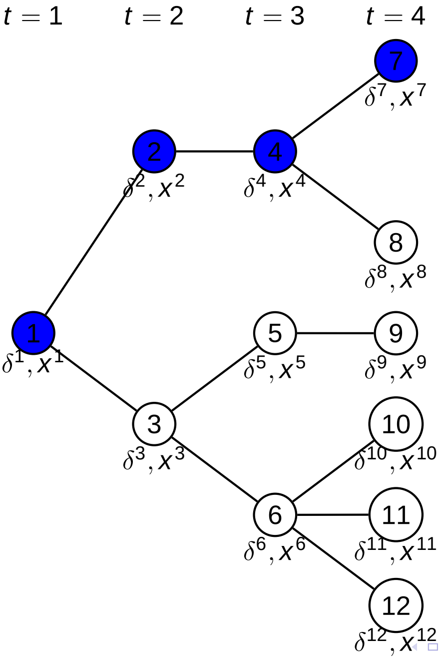
δ12, x12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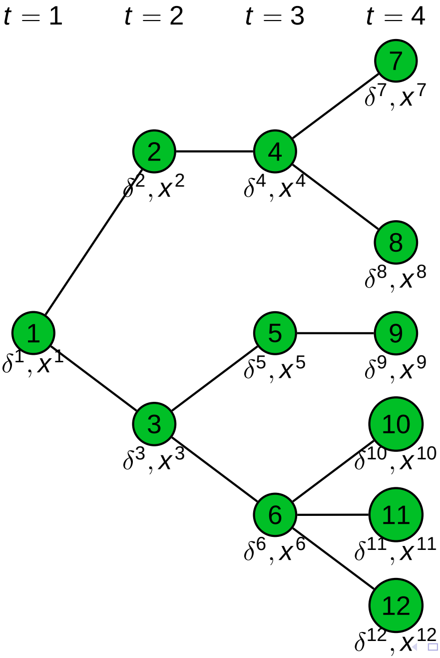
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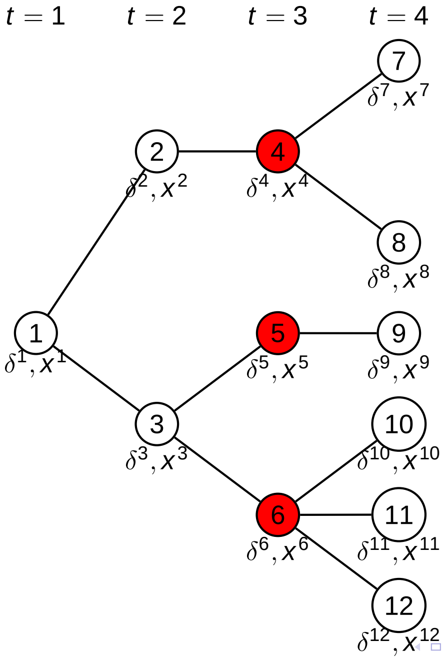

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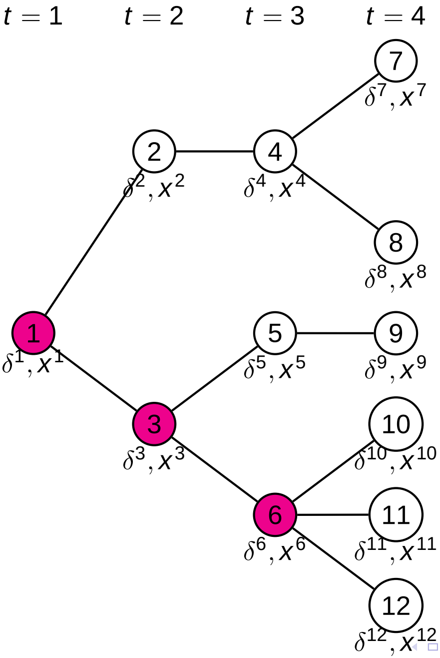
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Notation

- $\mathcal{T}$, set of the T stages along the horizon.
- Ω, set of scenarios.
- $\mathcal{G}$, set of scenario groups, so that we have a directed graph where $\mathcal{G}$ is the set of nodes.
- $\mathcal{G}^t$, set of scenario groups in stage t, for $t \in \mathcal{T}$ ($\mathcal{G}^t \subseteq \mathcal{G}$).
- Ω^g, set of scenarios in group g, for $g \in \mathcal{G}$ ($\Omega^g \subseteq \Omega$).
- $\tilde{\mathcal{A}}^g$, set of ancestor nodes (scenario groups) in the scenario tree to node (scenario) g (including itself), for $g \in \mathcal{G}$.
- $\mathcal{A}^g \subseteq \tilde{\mathcal{A}}^g$, set of ancestor nodes in the scenario tree to node g (including itself) whose related variables have nonzero elements in the constraints of node g, for $g \in \mathcal{G}$.

Notation (c.)

w^{ω}, likelihood or weight assigned by the user to scenario $\omega \in \Omega$.

w^{g}, weight assigned by modeler to scenario group $g \in \mathcal{G}$. It is computed as

$$w^{g} = \sum_{\omega \in \Omega} w^{\omega}$$

Note: Any scenario group g from last stage is a singleton one and, since $\omega \in \Omega^{g}$ for $g \in \mathcal{G}^{T}$, then let assume $g \equiv \omega$.

Multistage Mixed 0-1 DEM: Risk neutral strategy

$$
\begin{aligned}
z_{RN} &= max \sum_{g \in \mathcal{G}} w^g F^g(\delta^g, x^g) \\
\text{s.t.} & \sum_{q \in \mathcal{A}^g} f^g(\delta^q, x^q) = h^g \qquad \forall g \in \mathcal{G} \\
\delta^g &\in \{0,1\}^{n\delta(g)}, \quad x^g \in \mathbb{R}^{nx(g)} \quad \forall g \in \mathcal{G}.
\end{aligned} \tag{2}
$$

Time Stochastic Dominance risk averse strategy

- The risk neutral (RN) model maximizes the objective function expected value. It ignores the variability of the objective function value over the scenarios, in particular the “left” tail of the non-wanted scenarios.
- There are some risk averse approaches that additionally deal with risk management.
- Among them, the Time Stochastic Dominance (TSD) risk averse strategy reduces the risk of a negative impact of the solution in non-wanted scenarios (i.e., black swan scenarios).

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Time Stochastic Dominance risk averse strategy (c.)

TSD strategy also aims to maximize the objective function expected value as RN, but, additionally:

- A set of given thresholds on the value of each function under consideration (including the objective one) for each scenario should be satisfied
- with a bound *target* on the probability of failure due to a shortfall on reaching each threshold
- as well as a bound *target* on the expected shortfall on reaching it for each selected stage along the time horizon, and
- a bound *target* on the shortfall

TSD strategy: Elements

Set of **modeler-driven profiles**, say $\mathcal{P}^t \; \forall t \in \tilde{\mathcal{T}}^{\psi}$, $\psi \in \Psi$, being $\tilde{\mathcal{T}}^{\psi} \subseteq \mathcal{T}$ the set of stages where **TSD** has to be imposed for function $\psi \in \Psi$, where Ψ is the set of functions to consider in a decreasing order of priority.

- ϕ^p, objective function threshold to be satisfied up to scenario group g in the scenario tree, for $g \in \mathcal{G}^t$
- S^p, maximum shortfall *target* that is allowed on reaching the threshold up to scenario group g
- $\overline{s}^p$, upper bound *target* on the expected shortfall on reaching the threshold
- β^p, upper bound *target* on the fraction of scenarios with shortfall

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TSD strategy: Additional variables

- s^{gp}, shortfall (continuous) variable that, obviously, is equal to the difference (if it is positive) between threshold ϕ^p and the value in the given function up to scenario group g,
- ν^{gp}, 0-1 variable such that its value is 1 if there is shortfall for the value of the given function up to scenario group g. That is, $\nu^{gp} = 1$ for $s^{gp} > 0$.
- ϵ_S^p, $\epsilon_{\overline{s}}^p$ and ϵ_β^p, slack variables that take the violation of the S-, $\overline{s}$- and β-bounds, res.
- being M_S^p, $M_{\overline{s}}^p$ and M_β^p, big enough M-parameters for penalizing the slack variables in the objective function.

TSD strategy: SMINO $\rightarrow$ Sequential SMILO

$$z_{TSD} = max \sum_{g \in \mathcal{G}} w^g F_1^g(\delta^g, x^g) -$$

$$\sum_{\psi \in \Psi} \sum_{t \in \tilde{\mathcal{T}}^{\psi}} \sum_{p \in \mathcal{P}^t} (M_S^p \epsilon_S^p + M_{\overline{S}}^p \epsilon_{\overline{S}}^p + M_\beta^p \epsilon_\beta^p)$$

s.t.

$$\sum_{q \in \mathcal{A}^g} f^g(\delta^q, x^q) = h^g \qquad \forall g \in G$$

$$\delta^g \in \{0,1\}^{n\delta(g)},\ x^g \in \mathbb{R}^{nx(g)} \qquad \forall g \in \mathcal{G}$$

$$\sum_{q \in \tilde{\mathcal{A}}^g} F_\psi^g(\delta^q, x^q) + s^{gp} \geq \phi^p \qquad \forall g \in \mathcal{G}^t, p \in \mathcal{P}^t, t \in \tilde{\mathcal{T}}^{\psi}, \psi \in \Psi$$

$$0 \leq s^{gp} \leq S^p \nu^{gp} + \epsilon_S^p,\ \nu^{gp} \in \{0,1\} \qquad \forall g \in \mathcal{G}^t, p \in \mathcal{P}^t, t \in \tilde{\mathcal{T}}^{\psi}, \psi \in \Psi$$

$$\sum_{g \in \mathcal{G}^t} w^g s^{gp} \leq \overline{s}^p + \epsilon_{\overline{S}}^p \qquad \forall p \in \mathcal{P}^t, t \in \tilde{\mathcal{T}}^{\psi}, \psi \in \Psi$$

$$\sum_{g \in \mathcal{G}^t} w^g \nu^{gp} \leq \beta^p + \epsilon_\beta^p \qquad \forall p \in \mathcal{P}^t, t \in \tilde{\mathcal{T}}^{\psi}, \psi \in \Psi$$

$$\epsilon_S^p, \epsilon_{\overline{S}}^p, \epsilon_\beta^p \in \mathbb{R}_+ \qquad \forall p \in \mathcal{P}^t, t \in \tilde{\mathcal{T}}^{\psi}, \psi \in \Psi.$$

Risk Averse multistage Stochastic Dominance Constraints (SDC) strategies: Refs.

- First-order SDC: Gollmer-Neise-Schultz SIOPT'08 for two-stage.
- Second-order SDC: Gollmer-Gotzes-Schultz MP'11 for two-stage.
- Computational comparison of risk averse strategies: Alonso.Ayuso-Carvallo-LFE-Guignard-Pi-Puranmalka-Weintraub EJOR'14.
- TSD: On Time Stochastic Dominance induced by mixed integer-linear recourse in multistage stochastic programs: LFE-Garín-Merino-Pérez EJOR'14 1st revision.

Multistage decomposition methods

- **Lagrangeans** (MCLD strong lower bound provider), LFE-Garín-Unzueta-Pérez COR'13 and submitted 2014
- **Branch-and-Fix Coordination (BFC):**
  - **exact sequential BFC** risk neutral (LFE-Garín-Merino-Pérez COR'12)
  - **exact parallel computing BFC** risk neutral (Aldasoro-LFE-Merino-Pérez COR'13)
- **inexact ELP** risk neutral (Beltrán.Royo-LFE-Monge-Rodriguez.Revines COR'14)
- **Parallel computing SDP risk neutral** (Aldasoro-LFE-Merino-Monge-Pérez submitted 2014)
- plus treating the **cross scenario group constraints**:
  - **exact BFC risk averse TSD** (LFE-Garín-Merino-Pérez EJOR'14 1st revision)
  - **inexact SDP risk averse TSD** (LFE-Monge-Romero.Morales COR'14 1st revision)
  - **inexact FRC risk averse TSD** (LFE-Garín-Pizarro-Unzueta in preparation)

Case study for solution validation

Computational Characteristics:

- BFC-TSD decomposition algo in C++ (LFE-Garín-Merino-Pérez EJOR'14 1st revision)
- CPLEX v12.5 as an auxiliary LP/MIP solver
- SW/HW: WS Dell Precision T7600, LINUX (v. Debian2.6.32-48), 64 bits, Intel(R) Xeon(R) CPU E5-2630 @ 2.3 GHz, 12 Gb of RAM and 8 cores.
- Case study: Instance P7

Table 1: Risk Neutral model (2). Dimensions. Instance P7

m	$n\delta$	nx	nel	$dens$	$\lvert\Omega\rvert$	$\lvert\mathcal{G}\rvert$	T
14400	3456	10368	1206875	0.60	182	288	5

Table 2: Risk Neutral (2). Instance P7

T	C	z^{RN}_{BFC}	$z^{RN}_{CPX}(OG\,\%)$	$GG\,\%$	t^{RN}_{BFC}	t^{RN}_{CPX}
5	7	269441	269441(*)	0.0	63	345

(*): Optimality gap achieved ($<$ 0,01 %)

- Solution value of RN model (2):
  Expected profit z_{RN}=269441.
- Modeler-driven thresholds: $\phi^p = \delta^p \times z_{RN}$ and, so, ϕ^1=255969, ϕ^2=242497, ϕ^3=229025 and ϕ^4=215553.
- Computed number of scenarios with shortfall on reaching ϕ^p out of 182 scenarios in RN sol. $|\Omega^1_{RN}|$=50, $|\Omega^2_{RN}|$=22, $|\Omega^3_{RN}|$=6 and $|\Omega^4_{RN}|$=0.
- Computed average shortfall: $\overline{s}^1_{RN}$=3498.9, $\overline{s}^2_{RN}$=886.9, $\overline{s}^3_{RN}$=56.6, $\overline{s}^4_{RN}$=0.0.

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Looking for acceptable risk reduction on the RN results

Assuming that the RN picture is carrying out an excessive profit risk, let us consider that it has been decided that:

- Upper bound on number of scenarios with shortfall should be reduced from 50 to $|\Omega^1|$=45, from 22 to $|\Omega^2|$=17, from 6 to $|\Omega^3|$=3 and the zero scenario policy is kept for profile 4 (i.e., $|\Omega^4|$=0).
- Upper bound on related expected shortfall should be reduced from 3498.9 to $\overline{s}^1$=3240.0, from 886.9 to $\overline{s}^2$=650.0, from 56.6 to $\overline{s}^3$=35.0 and the zero shortfall policy is obviously kept for profile 4 (i.e., $\overline{s}^4$=0.0).

Table 3: TSD model (3). Dimensions for 4 profiles, $\tilde{\mathcal{T}} = \{T\}$

Strategy	m	$n\delta$	nx	$n\nu$	n_{ϵ_β}	ns	$n_{\epsilon_{\overline{S}}}$	$dens$	$\lvert\Omega\rvert$	$\lvert\mathcal{G}\rvert$	T
FSD	15132	3456	10368	728	4	0	0	0.628	182	288	5
SSD	15132	3456	10368	0	0	728	4	0.628	182	288	5
TSD1	15864	3456	10368	728	4	728	4	0.571	182	288	5

Table 4: TSD risk averse strategy (3). $\tilde{\mathcal{T}} = \{T\}$. Instance P7

$p \in \mathcal{P}^T = \{1,2,3,4\}$ $\beta^p(\lvert\Omega^p\rvert)$	$\overline{s}^p$	$p \in \mathcal{P}^{T-1} = \{1,2\}$ $\beta^p(\lvert\Omega^p\rvert)$	$\overline{s}^p/\phi^p$	z_{BFC}^{TSD1}	$z_{CPX}^{TSD1}(OG\,\%)$	GG %	t_{BFC}^{TSD1}	t_{CPX}^{TSD1}
0.248(45)	3240	0.495(90)	5839	269247	269273 (*)	*	77	3135
0.094(17)	650	0.181(33)	1063					
0.017(3)	35	$\phi^1 = 67360.2$						
0(0)	0	$\phi^2 = 53888.2$						

(*): Optimality gap achieved ($< 0{,}01\,\%$)
*: Goodness gap achieved ($< 0{,}01\,\%$)

- Solution value of TDD1 model (3): Let us consider the expected profit z_{CPX}^{TSD1}=269273 obtained by plain use of CPLEX
- (Notice that the BFC-TSD algorithm gives a close value, z_{BFC}^{TSD1}=269247 whose goodness gap is $< 0{,}01\,\%$).
- Observe that the expected profit of the risk neutral model (2), z_{RN}=269441 has only been slightly reduced by the risk averse model TSD (3) and, on the other hand, the profit risk of non-wanted scenarios has been reduced to the modeler-driven bounds.

Rationale behind Table 4

Risk averse TSD model (3) for $\tilde{\mathcal{T}} = \{T\}$.

- It only considers last stage, T (i.e., TSD1 policy is used).
- However, as an example, a modeler-driven non-wanted profit threshold ϕ^p at stage $T-1$ is shown in the table for $p = 1, 2$ in case that the TSD1 solution is accepted. It is also shown (in color blue);
- $\beta^p(|\Omega^p|)$: Computed fraction of scenarios (number of scenarios) in set Ω with shortfall on reaching profit threshold $\phi^p \, \forall p \in \mathcal{P}^{T-1} = \{1, 2\}$
- $\overline{s}^p$: Computed average shortfall.

Those values *have been computed from the TSD1 solution* for the profit obtained up to stage $T-1$, say

$$z_{T-1} = \sum_{g \in \mathcal{G}^{T-1}} w^g \sum_{q \in \tilde{\mathcal{A}}^g} (a^q \delta^q + b^q x^q).$$

Looking for acceptable risk reduction on TSD1 results

Assuming that the TSD1 picture is carrying out an excessive profit risk up to stage $T-1$, let us consider that it has been decided that:

- Keep the β^p- and $\overline{s}^p$-bounds on risk reduction for last stage T as they are for TSD1 strategy, for $p \in \mathcal{P}^T$.
- Upper bound on number of scenarios with shortfall up to stage $T-1$ should be reduced from 90 to $|\Omega^1|$=80 and from 33 to $|\Omega^2|$=26.
- Upper bound on related expected shortfall should be reduced from 5839.0 to e^1=5400.0 and from 1063.0 to e^2=800.0.

- Rationale behind Tables 5 and 6: Risk averse TSD model (3) for $\tilde{\mathcal{T}} = \{T-1, T\}$, where $|\mathcal{P}^T| = 4$ and $|\mathcal{P}^{T-1}| = 2$ (so-named TSD2).

Table 5: TSD model (3). Dimensions for 4 profiles, $\tilde{\mathcal{T}} = \{T\}$ and 6 profiles, $\tilde{\mathcal{T}} = \{T-1, T\}$

Strategy	m	$n\delta$	nx	$n\nu$	n_{ϵ_β}	ns	$n_{\epsilon_{\overline{S}}}$	$dens$	$\|\Omega\|$	$\|\mathcal{G}\|$	T
FSD	15132	3456	10368	728	4	0	0	0.628	182	288	5
SSD	15132	3456	10368	0	0	728	4	0.628	182	288	5
TSD1	15864	3456	10368	728	4	728	4	0.571	182	288	5
TSD2	16164	3456	10368	876	6	876	6	0.561	182	288	5

Table 6: TSD risk averse strategy (3). $\tilde{\mathcal{T}} = \{T-1, T\}$. Instance P7

$p \in \mathcal{P}^T = \{1,2,3,4\}$ $\beta^p(\lvert\Omega^p\rvert)$	$\overline{s}^p$	$p \in \mathcal{P}^{T-1} = \{1,2\}$ $\beta^p(\lvert\Omega^p\rvert)$	$\overline{s}^p$	z_{BFC}^{TSD2}	$z_{CPX}^{TSD2}(OG\,\%)$	GG %	t_{BFC}^{TSD2}	t_{CPX}^{TSD2}
0.248(45)	3240	0.440(80)	5400	269204	269222 (0.01 %)	*	225	–
0.094(17)	650	0.143(26)	800					
0.017(3)	35							
0(0)	0							

(*): Optimality gap achieved ($<$ 0,01 %)
*: Goodness gap achieved ($<$ 0,01 %)
–: Out of memory (12Gb) or time limit (6h) exceeded

- Solution value of TDD2 model (3): Let us consider the expected profit z_{CPX}^{TSD2}=269222 obtained by plain use of CPLEX
- (Notice that the BFC-TSD algorithm gives a close value, z_{BFC}^{TSD2}=269204 whose goodness gap is $<$ 0,01 %).
- Observe that the expected profit of the TSD1 risk averse model (3), z_{TSD1}=269273 has only been slightly reduced by the risk averse model TSD2 (3) and, on the other hand, the profit risk of non-wanted scenarios has been reduced to the modeler-driven bounds.

Successful results: Production planning

Computational Characteristics:

- Parallel Compting: MPI, Message Passing Interface.
- CPLEX v12.5
- Experimental Parallel-SDP code in C (Aldasoro-LFE-Merino-Perez COR'13).
- ARINA computational cluster, SGI/IZO-SGIker at UPV/EHU,

  We have used 8 xeon nodes, where each has 12 processors and 48Gb of RAM, 2.4GHz (96 total processors).
- Realistic instances from Cristobal-LFE-Monge COR'09.

- $\mathcal{T}$=16 periods, $\mathcal{E}$=3 stages, Randomly generated Ω=7766 scenarios
- *m*=4.25 Million cons, *n*01=1.16 Million 0-1 varis, *nc*=2.7 Million continuous vars
- S-SDP: 22167 secs, number of *nprob*=3249 MILP subproblems
- P-SDP: $GG = 0{,}16\,\%$ optimality gap versus plain use CPLEX v.12.5, elapsed time=3442 secs, Effciency=53.67 %
- CPLEX: running out of memory (35Gb) after 5926 secs, solution value with OG=1.80 % quasi-optimality gap at stopping time instant.

- $\mathcal{T}$=16 periods, $\mathcal{E}$=4 stages, Randomly generated Ω=15435 scenarios
- CPLEX: Stop due to out of memory (35Gb), no LP feasible solution at 3003 secs.
- ***m*=57.8 million cons, *n*01=15.4 million 0-1 vars, *nc*=38.5 million continuous vars**
- S-SDP: 26180 secs, number of *nprob*=517 MILP subproblems.
- P-SDP: 2446 secs, Effciency=89.19.

Summary. Instance c85

- $\mathcal{T}$=16 periods, $\mathcal{E}$=4 stages, Randomly generated Ω=15435 scenarios
- CPLEX: Stop due to out of memory (35Gb), no LP feasible solution at 3003 secs.
- ***m*=57.8 million cons, *n01*=15.4 million 0-1 vars, *nc*=38.5 million continuous vars**
- S-SDP: 26180 secs, number of *nprob*=517 MILP subproblems.
- P-SDP: 2446 secs, Effciency=89.19.

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